Investment Decisions, Voluntary Disclosure, Myopia, and Bounded Inefficiency.*

Ilan Guttman† Ilan Kremer‡ Andrzej Skrzypacz§ Elyashiv Wiedman¶

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Abstract

We present a strategic voluntary disclosure model where a manager chooses between two projects that she is privately informed about their values. Only one of the projects generates a signal that can be credibly disclosed in the short term. Absent managerial disclosure, the stock price reflects suspicions that the manager is concealing a negative project value. Consistent with the existing literature, we find that due to short-term price incentives, strategic disclosure may lead to inefficiency in selecting the project whose value can be disclosed, even though it is the inferior of the two projects. However, our main result demonstrates that the inefficiency is quite limited, even when the manager cares almost entirely about the short-term stock price.

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†Stern School of Business, New York University (e-mail: iguttman@stern.nyu.edu);

‡Department of Economics, School of Business Administration, and Federmann Center for the Study of Rationality, The Hebrew University of Jerusalem, and University of Warwick (e-mail: ikremer@huji.ac.il).

§Graduate School of Business, Stanford University (e-mail: skrz@stanford.edu).

¶School of Business Administration, The Hebrew University of Jerusalem (e-mail: e.wied@mail.huji.ac.il).

1 Introduction

Consider a manager of a public firm who must choose between two projects, A and B. The manager is entrusted with this decision-making right because she possesses private information about the returns of these projects, and shareholders hope that she will make the efficient choice. However, if the manager cares about short-term stock prices and the two projects differ in terms of the amount of information she can disclose in the short term, the manager may not optimize for the long-term value of the firm. In this paper, we examine how voluntary disclosure creates pressure to disclose short-term information and to what extent that pressure can lead to inefficient project selection. Our main finding is that such inefficiency is limited.

We present a model based on investment choice and strategic information disclosure.[1] At time $t = 1$, the manager observes the expected returns, a, and b, of two mutually exclusive projects, A and B, and must decide which one to implement. If she chooses A, she can immediately, costlessly, and verifiably disclose a. However, the manager can also temporarily conceal the value of project A, postponing the revelation of the cash flow until $t = 2$. If the manager chooses project B, she cannot disclose b in the short term. This captures scenarios such as project A is short-term and project B is long-term; hence, the information about the cash flows realized from A can be disclosed sooner. Alternatively, the two projects may vary in terms of proprietary disclosure costs, with project A having no disclosure costs and project B having sufficiently high costs (due to adverse use of this information against the firm by various stakeholders, e.g., competitors, suppliers, customers, regulators).

At $t = 1$, investors do not directly observe the project choice but observe the manager's disclosure or lack thereof. The firm is priced according to investors' expectations of the chosen project's value. The manager cares about both the firm's short-term price and its long-term fundamental value. Specifically, she assigns a weight of w to the long-term fundamental value and $(1 - w)$ to the short-term price. The equilibrium is based on rational expectations, so prices reflect all the information that becomes public. In particular, if the manager remains silent at $t = 1$, the stock price equals the project's expected value. We refer to this price as the "silence price."

[1] For recent studies on the interaction of investment choice and voluntary information disclosure, see Ben-Porath, Dekel, and Lipman (2018) and Guttman and Meng (2021).

The silence price accounts for the possibility that the manager either chose project A (the short-term project or the one with low disclosure costs) and strategically concealed it due to its low value or opted for project B for which there is no verifiable evidence to disclose (or the evidence would be too costly to disclose). At $t = 2$, the cash flow of the chosen project is revealed, and the firm price equals this liquidating value.

When the manager is somewhat myopic ($w < 1$), the manager might be tempted to choose project A for which she can reveal short-term information, even if that project has a lower value. This happens because if the manager chooses project B (and thus cannot disclose any immediate information), investors may suspect that she is hiding the selection of a low-value project A. Such suspicions can negatively affect the stock price when no disclosure is made. Consequently, this may cause the manager to opt for project A inefficiently. This implies that full efficiency cannot be supported in equilibrium.

When the manager is fully myopic ($w = 0$), the pressure to make an early disclosure can become extreme. As we show, in this case, there exists an equilibrium where the manager always selects project A, and the silence price reflects the worst possible outcome of project A. We refer to this equilibrium as the "unraveling equilibrium," because the manager always discloses the project's outcome. In this equilibrium, the efficiency loss can be quite significant.

One may expect similar inefficiency for almost myopic managers, i.e., when w is close to 0. However, we show that this is not the case. The level of inefficiency for any $w > 0$ is quite limited. We argue that one should not expect the inefficiency to be that extreme.

First, we show that in the more realistic case where the manager is not fully myopic ($w > 0$), the equilibrium inefficiency is guaranteed to be much smaller than in the unraveling equilibrium. Second, even when the manager is fully myopic ($w = 0$), one should not necessarily expect the equilibrium to be as inefficient as the unraveling equilibrium. In that case, the equilibrium is not unique, and there are more efficient equilibria. In particular, any limit of equilibria as $w \to 0$ remains an equilibrium for $w = 0$, and any such limiting equilibrium is significantly more efficient than the unraveling equilibrium.

Our main results can be described as follows:

- We first consider an example where both projects are uniformly distributed over $[0, 1]$. We show that in the unraveling equilibrium, there is an efficiency loss of 25% (relative to the first-best project selection). In contrast, this loss reduces to 2.27% in the limiting equilibrium and is even smaller for any $w > 0$.[2]

- We show that when we focus on the least efficient equilibrium, efficiency increases with the manager's weight on the long-term value, w.

- Assuming the distributions are log-concave, as the manager becomes more myopic ($w \to 0$), the equilibrium set converges to a unique equilibrium. This is the most efficient equilibrium in the case of $w = 0$.[3]

We provide two bounds that contrast the efficiency of the limiting equilibrium with that of the unraveling equilibrium. These bounds apply to arbitrary distributions of the two projects over $[0, 1]$, which need not be identical. Moreover, these bounds are applicable even if the limiting equilibrium is not unique.

- Given that in the unraveling equilibrium, the manager always selects project A, this leads to a surplus of $\mathbb{E}(A)$. We show that the surplus of the limiting equilibrium is at least $\mathbb{E}(max\{A, \mathbb{E}(B)\})$.

- We show that in the unraveling equilibrium, the manager may eradicate the entire surplus. In contrast, the limiting equilibrium outcome preserves more than 50% of the total surplus. In other words, the difference between these two equilibria is more significant than the difference between a limiting equilibrium and a fully efficient one.

The intuition for the equilibrium surplus to be at least $\mathbb{E}(max\{A, \mathbb{E}(B)\})$ is as follows. First, when the manager cares even slightly about the firm's long-term value, no matter what the silence price is, if a is very low and b is very high, the manager would select B and remain silent. Second, when the agent chooses to be silent, she

[2] We show that for any $w > 0$ the equilibrium is unique.

[3] The uniform distribution is log-concave, so the limiting equilibrium described above is the unique limit when $w \to 0$.

chooses efficiently. Therefore, the silence price has to be at least $\mathbb{E}(B)$. Finally, consider the limiting case where $w \to 0$ so that the manager maximizes the short-term price. When the silence price is at least $\mathbb{E}(B)$, the manager can guarantee herself a payoff of at least $\mathbb{E}(max\{A, \mathbb{E}(B)\})$ by choosing and disclosing A when it is above the silence price. Since the equilibrium price is, on average, correct, this is the lower bound on the expected value of the chosen project. As we later show, if the manager cares more about the firm's long-term value (w increases), her choices become even more efficient, and the silence price increases.

It has long been argued that managers often prioritize short-term profits at the expense of long-term value. The extant literature studied multiple settings in which short-term (myopic) incentives lead to inefficiencies, where for high degrees of myopia, the magnitude of inefficiencies can become very large (unbounded) (e.g., Stein 1989). Our model demonstrates that voluntary disclosure with project choice can lead to inefficiencies. Yet, the inefficiencies due to myopic incentives are quite limited, even when the level of myopia goes to infinity. In other words, we show that differences in the possibility of disclosing information combined with managerial myopia create incentive problems. Still, the resulting inefficiency, relative to the first best investment, is quite small.

The empirical literature regarding the effect of managers' short-termism provides mixed results. One strand of the literature finds evidence of efficiency loss.[4] On the other hand, others point out that short-termism is not a first-order effect.[5] Despite the mixed empirical support, the claim that short-termism is an important concern is widespread. This is due to the intuitive theoretical appeal of this claim.[6] This concern supports measures such as the adoption of dual-class share structures to grant managers autonomy against short-term pressures or the reluctance of private firms to go public.

[4] For example, Bernstein (2015) found that public firms become less innovative. Kraft, Vashishtha, and Venkatachalam (2018) argued that when firms are required to provide more frequent financial reports, it negatively affects their investments.

[5] For example, Roe (2021) suggested that managers' short-termism has no significant effect on investment in R&D, and Kajüter, Klassmann, and Nienhaus (2019) studied the effect of increasing reporting requirements but did not find evidence of myopic investments.

[6] For example, Joe Biden, during the presidential campaign in 2016, wrote "Short-termism—the notion that companies forgo long-run investments to boost near-term stock price—is one of the greatest threats to America's enduring prosperity." See Joe Biden at the WSJ, September 27, 2016: https://www.wsj.com/articles/how-short-termism-saps-the-economy-1475018087.

In this paper, we propose an explanation for why myopia may not always lead to significant inefficiency in practice. We argue that when myopia is driven by concerns about voluntary disclosure, the resulting inefficiency may be limited. Our analysis, consistent with prior literature, demonstrates that while markets efficiently price firms based on the available information, investment inefficiencies can arise when managers exhibit some degree of myopia. However, our primary finding suggests that the extent of investment inefficiency remains quite limited when managers exhibit even minimal concern for long-term fundamental value. Notably, substantial investment inefficiencies are observed only when managers focus exclusively on short-term stock prices and when considering the worst possible equilibrium. In other words, we suggest that a significant portion of investment inefficiency can be mitigated when managers have even marginal long-term incentives. [7]

1.1 Related literature

Narayanan (1985) demonstrated how managers, driven by career concerns, may take suboptimal early actions to enhance their reputation rather than pursuing more profitable strategies over time. Stein (1989) examined a signaling model in which a manager may borrow, at some cost, some of the firm's future profits to inflate current earnings artificially. He illustrated how managers, even with slight myopic incentives, inflate current earnings at the expense of future value despite investors' rational anticipation of this behavior. Our model also demonstrates that myopic managers invest inefficiently in projects. However, we show that the inefficiencies resulting from managerial myopia are quite limited.[8] This key difference stems from our focus on the voluntary nature of disclosure, where the pressure is for managers to present results, as otherwise, they are viewed as potentially concealing negative realizations.

The empirical literature on the effect of managerial myopia on corporate investment is extensive. It is important to note that measuring managerial myopia poses challenges; thus, direct examination of the question at hand is limited. The em-

[7] Of course, if managers have contracts that align their incentives with long-term investors (or are aligned for other reasons), this would reduce the inefficiency of their investment decisions. Our contribution is to show that, in our model, even in the absence of strong alignment, managers make efficient decisions most of the time.

[8] In Stein's model, if an almost fully myopic manager could shift earnings at a low cost, she would eradicate the entire surplus.

pirical literature yields mixed results regarding the effect of managerial myopia on investment. In a survey of CEOs, Poterba and Summers (1995) found that managers prefer investing in short-term projects rather than long-term ones due to stock market pressures. Many surveyed CEOs argue that they would allocate more investment toward long-term projects if stock market participants would properly value these investments. Some studies examine the differences in investments and innovations between public and private firms, assuming that public firms are more prone to myopia. Asker, Farre-Mensa, and Ljungqvist (2015) found that public firms invest less than private firms and are less responsive to changes in investment opportunities, particularly for firms whose stock prices are more sensitive to earnings. In a similar vein, Ladika and Sautner (2020) exploited a quasi-experiment that led executives to become more myopic, resulting in a reduction of their investments. In contrast, Acharya and Xu (2017) found that public firms exhibit greater innovation compared to private firms in industries reliant on external financing, whereas the disparity is less pronounced in industries where internal financing prevails. Similarly, Gilje and Taillard (2016) found that public firms invest more in response to positive investment opportunities than private firms, and access to external financing is a key driver of this difference. Thus, it remains ambiguous whether the variation between public and private firms stems from managerial myopia or differences in their funding opportunities.

The impact of financial reporting serves as a crucial lens through which to explore the influence of myopia on investment decisions. In a survey of 400 executives, Graham, Harvey, and Rajgopal (2005) studied the effect of reporting and disclosure on various managerial decisions. Seventy-eight percent of the surveyed executives admitted that they sacrifice long-term value to boost their firms' current earnings. Gigler, Kanodia, Sapra, and Venugopalan (2014) theoretically examined the effect of the frequency of required financial reporting on investment decisions. They showed that more frequent reporting reduces inefficient investment in negative NPV projects but may increase investment in less profitable projects that yield better results in the short term. Edmans, Heinle, and Huang (2016) showed that information disclosure requirements induce managers to invest more in projects that generate more hard information than soft information, resulting in better financial efficiency but also leading to real efficiency loss. Kraft, Vashishtha, and Venkatachalam (2018) used changes in reporting requirements in the US from 1950 to 1970 and showed that an increase in reporting frequency leads to a decline in investments. In contrast, Kajüter, Klassmann, and Nienhaus (2019) studied a regulation change (regression discontinuity) in the reporting frequency for some firms but did not find any evidence of myopic investments.

In our model, managers may invest inefficiently by selecting short-term projects over long-term projects due to the pressure to boost their firm's short-term market price. Our finding of the limited impact of myopia on investment efficiency, especially when the manager has slight long-term incentives, may explain why some empirical studies find an effect of myopia on investment inefficiency while other studies fail to identify this effect.

Our model is also pertinent to recent theoretical literature examining the relationship between voluntary disclosure and managers' investment decisions. We assume that the manager learns the values of both short- and long-term projects before making an investment decision. Furthermore, the manager may decide to remain silent when choosing a short-term project, pretending to pursue the long-term one for which she is unable to report results.

The closest paper to ours is Ben-Porath, Dekel, and Lipman (2018). In their paper, an agent selects at $t = 0$ between two projects, F and G, that will be realized and may be disclosed at $t = 1$.[9] They also consider the inefficiency of project choice and present two results related to ours. They show in Theorem 2 that when the two projects share the same likelihood of generating verifiable evidence, the inefficiency is bounded by 1/2. However, their Theorem 7 is closer to our setting, as it considers the case where the two projects generate verifiable evidence with different probabilities. They show that the inefficiency can be as high as $1 - w$ (note, they denote w by α). In other words, as the agent places less weight on the fundamental value of the project, inefficiency can become arbitrarily high. This is in sharp contrast with our main result where inefficiency is bounded by 1/2 across all distributions.

Several major differences distinguish our paper from Ben-Porath, Dekel, and Lipman (2018) and explain the difference in results. One main distinction lies in the

[9]Ben-Porath, Dekel, and Lipman (2018) analyze disclosure by an agent that may face additional disclosure by a challenger. When there is no challenger their game is similar to ours.

agent's information advantage when making a selection. In our model, the principal (i.e., investors) prefers to delegate the decision to the manager because the manager has superior knowledge about which project is better. E.g., in the uniform distribution example, without delegation investors can expect a payoff of 1/2 while delegation yields a payoff closer to the first-best of 2/3. In contrast, in Ben-Porath, Dekel, and Lipman (2018), the agent and the principal share the same information at the project selection stage, meaning the principal would rather retain decision-making authority instead of delegating it. Another fundamental difference concerns the primary economic force driving the results in Ben-Porath, Dekel, and Lipman (2018). In their model, the agent favors riskier projects to exploit the option of concealing unfavorable outcomes. This mechanism is absent in our setting. Finally, in the equilibrium of Ben-Porath, Dekel, and Lipman (2018), the principal always knows which project the agent has chosen. However, in our model, uncertainty remains when the manager remains silent, leaving the principal unaware of which project was selected.

Finally, in terms of methodology, our paper contributes to the literature on robust predictions of economic models. The two most related strands are the literature on robust mechanism design and the literature on the price of anarchy. In that first strand, Carroll (2015) explores optimal contracts where the set of projects an agent has is only partially known to the principal. Our analysis of the bounds on inefficiency over a set of possible distributions reflects the analysis in Kang, Pernice, and Vondrák (2022) that considers the deadweight loss of mechanisms across a set of distributions within a bilateral monopoly context. For a review of robustness in mechanism design, see Carroll (2019). In parallel, akin to our study, the literature on the price of anarchy evaluates the ratio of the worst equilibrium payoffs to the efficient payoffs. For a survey of that literature, see Nisan, Tardos, Roughgarden, and Vazirani (2007) (specifically, Chapter 17).

2 Model

Our model describes a manager who chooses between two projects and risk-neutral market/investors who price the firm.

There are two periods, $t \in \{1, 2\}$, as illustrated in Figure 1. At time $t = 1$, the manager privately observes information about two mutually exclusive projects, A and B, and chooses which to pursue. A is a project that generates a signal that can be credibly and costlessly disclosed in the short term, while B is a project that generates such a signal only in the long term. The information the manager observes is the (expected) value/cash flow of each project. We denote these values by a and b, respectively.[10] These are realizations of independent random variables A and B drawn according to atomless distributions $F_A(a)$ and $F_B(b)$ with densities $f_A(a)$ and $f_B(b)$, and common support normalized to $[0, 1]$.[11]

At $t = 1$, after choosing which project to pursue, if the manager selects A, she can choose to disclose its value, a, immediately. Disclosure is costless and truthful/verifiable. If she opts for project B, she has no verifiable information to disclose at $t = 1$, nor can she credibly convey that she chose project B.

At $t = 2$, the value of the chosen project, V, becomes public: it is equal to a if the manager chose A, and to b if she chose B.[12]

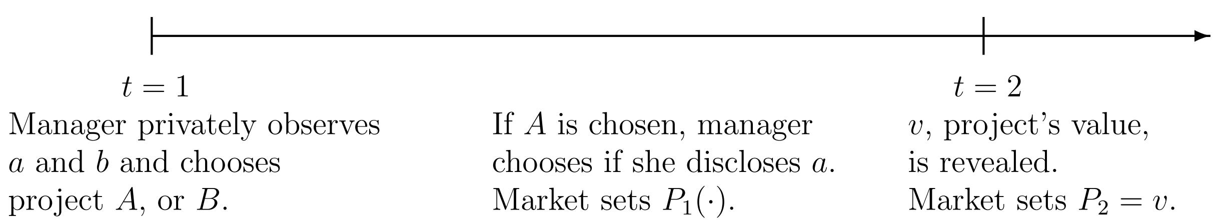

Figure 1: Timeline of events.

[10]The manager does not need to have perfect foresight at $t = 1$. The following setting yields similar results. Conditional on choosing project A, the value is $V = a + \epsilon_a$, and conditional on choosing project B, the value is $V = b + \epsilon_b$, where ϵ_a and ϵ_b are mean-zero random variables. In this interpretation, a and b are the expected values of the two projects observed by the manager.

[11]Note that we identify the project names by the names of the random variables: A, B denote both the projects' names and the random variables which are values of those projects.

[12]An alternative representation of our situation is that the short-term project A generates an immediate profit of a at $t = 1$, while the long-term project B, if selected, yields a profit of b only at $t = 2$. However, if the manager pursues the short-term project A, she can postpone the recognition of its profit until $t = 2$, making it appear as if she chose the long-term project. In other words, even if the manager chooses the short-term project, she can delay disclosing her information, making it indistinguishable from choosing a long-term project.

The manager cares about short-term and long-term prices, denoted by P_1 and P_2, respectively.[13] The manager maximizes her expected utility, which is a weighted sum of the short-term and long-term prices:

$$U = (1 - w) \cdot P_1 + w \cdot P_2,$$

where $w \in [0, 1]$ is the commonly known weight that the manager assigns to the long-term valuation, P_2. If $w = 0$, the manager is fully myopic, i.e., she only cares about short-term market prices. Conversely, if $w = 1$, she only cares about the firm's long-term value.

In the long term, at $t = 2$, the firm's value is determined by the realized value of the implemented project, and thus $P_2 = v$. In the short term, at $t = 1$, the market/investors price the firm based on the expected value of the selected project and conditional on any available information. It is important to note that investors do not necessarily observe the selected project. Therefore, P_1 reflects the manager's strategy, particularly the disclosure of a or the lack thereof.

Remark 1. *We have presented the model where the manager has to choose between two projects to clarify the exposition. However, our model immediately generalizes to a manager having a choice between some number of type A projects and some number of type B projects that are mutually exclusive. The number of those projects can be random. Upon choosing a type A project, the manager will choose the best among them. Similarly, conditional on selecting a type B project, she will choose the best among those. To cover that generalization, we only need to reinterpret the distributions F_A and F_B as the distributions of the first-order statistic of the value of the projects of each type.*

Equilibrium

The equilibrium concept is based on the manager's strategy being optimal given market prices, and market prices being consistent with the manager's strategy and Bayes' rule. The manager's strategy comprises two components: (i) a selection rule

[13] Note that we do not explicitly model the manager incentives scheme but assume it is exogenously given.

$x(a,b) \in \{A, B\}$ that determines which project to pursue based on the values of the projects she observes, a and b, and (ii) if project A is selected, a decision of whether to disclose the value a at $t = 1$ or to delay it to $t = 2$.

Definition 1. *A Perfect Bayesian Equilibrium is a strategy of the manager and market prices such that:*

1. *(Manager Rationality) The manager's strategy maximizes her expected utility given the market prices* $\{P_1, P_2\}$.

2. *(Consistency of Market Prices). The price* P_1 *equals the expected value of* V *conditional on the available information and the manager's strategy. The price* P_2 *equals the value of* V.

Since we presented the equilibrium using a verbal description, it's pertinent to note several observations.

The equilibrium price at $t = 2$ is always $P_2 = v$, as the realized value of the chosen project is publicly revealed at that time. Regarding the $t = 1$ price, P_1, it necessitates consideration of two scenarios. If the manager chooses A and discloses a, then $P_1 = a$ as we assume that the disclosed information is verifiable and therefore credible. However, a more intricate scenario arises when the manager opts for silence. This silence may stem from either selecting A and concealing its realization, a, or opting for B, rendering disclosure infeasible. In instances where the manager remains silent at $t = 1$, we call the price the *silence price* and denote it by:

$$P^s = E[V|\text{silence}].$$

This conditional expectation is based on the investors' conjecture of the manager's strategy. In particular, what they can infer about V from managerial silence.

Given the silence price, P^s, the manager's best response is:

- Whenever the manager selects A she discloses a if and only if $a > P^s$.

- If $U(B) < U(A)$ the manager chooses A; and if $U(B) > U(A)$ the manager chooses B. In particular, for $w > 0$, the manager's optimal selection strategy is given by:[14]

$$x(a,b) = A \text{ if } \begin{cases} b < a & \text{or} \\ P^s < a < b \;\;\&\;\; (1-w)P^s + wb < a. \end{cases}$$

$$x(a,b) = B \text{ if } \left\{ a < b \;\;\&\;\; (1-w)P^s + wb > a. \right.$$

Note that inefficiency is embodied in the manager's best response when she is somewhat myopic, $w < 1$; sometimes she chooses A even though $a < b$. This inefficient choice occurs because choosing B constrains the short-term price to P^s, but choosing A allows disclosure and a short-term price of a. Inefficiency may occur only when $P^s < a < b$. The manager's project choice and the derived inefficiency are illustrated in Figure 2:

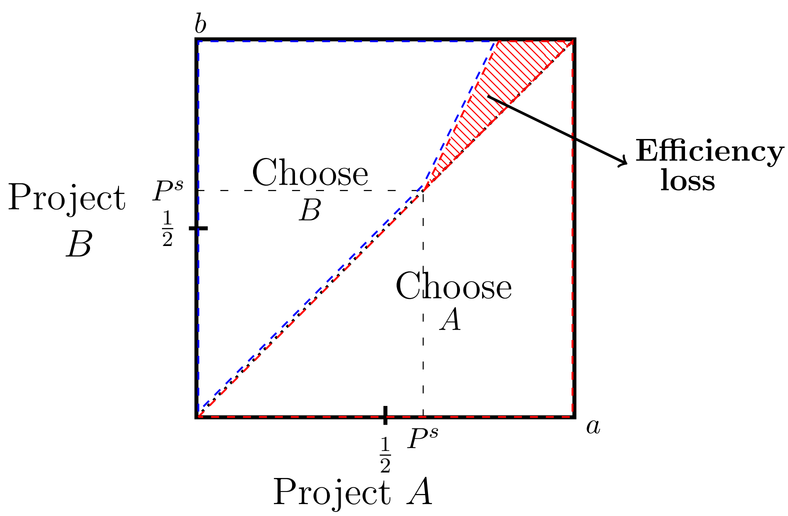

Figure 2: Project choice when A and B are uniformly distributed, and $w = \frac{1}{2}$. The patterned-lined area at the top-right corner represents combinations of a and b for which the manager selects project A, even though it is inferior to B.

Let $H(p)$ denote the expected value of V given that the manager remains silent at $t = 1$ and optimally responds to some exogenously specified silence price p. Since for

[14] In case $U(A) = U(B)$, the manager is indifferent. For $w > 0$, such indifference happens for a zero-measure set of realizations of (a, b), and how it is resolved does not affect the equilibrium. For $w = 0$, this indifference can happen for a positive measure of (a, b), leading to a multiplicity of equilibria.

$w > 0$ the manager's best response is unique, $H(p)$ is well defined. The equilibrium silence price is, therefore, any fixed point of H:

$$H(P^s) = P^s.$$

3 Example - Uniform Distribution

To provide intuition for the main results in the most straightforward manner, we analyze the model with uniform distributions of the value of the projects in this section. We start with this example because it allows us to solve for the equilibrium and the welfare loss analytically. Moreover, the graphical representation of the equilibrium strategies and welfare loss is very convenient, allowing us to illustrate the key insights below. In Section 4, we analyze the case of general distributions and demonstrate that many of the equilibrium properties under the uniform distributions hold true more generally.

Assume that the values of each of the projects are drawn independently from uniform distributions. To simplify the exposition we assume that $A, B \sim U[0, 1]$, implying prior means of $\frac{1}{2}$.

We establish the following lemma regarding the equilibrium in this scenario:

Lemma 1. *In the case of the uniform distribution, the equilibrium is unique for any $w > 0$.*

When the manager is fully aligned with investors' long-term considerations, i.e., $w = 1$, she always chooses efficiently, i.e., chooses the project with the higher value. We denote this scenario as the "first-best." The expected value of the implemented project under the first-best project choice is given by:

$$\mathbb{E}[v|v = Max\{a, b\}] = \int_0^1 z \cdot 2z dz = \frac{2}{3}.$$

A manager's strategy of selecting the project with the higher value, $Max\{a, b\}$, and disclosing the value whenever project A is selected (and remaining silent if choosing project B) yields a silence price of $\frac{2}{3}$.

Note that this strategy is not sustainable in equilibrium for any $w < 1$ for two reasons. First, a manager who chooses project A with a value below the silence price would prefer to remain silent over disclosing - to maximize her short-term payoff. Second, in certain cases, the manager may opt for project A even if its value is lower than that of project B, as the short-term gain from disclosing a value of a that is higher than the silence price outweighs the long-term loss. Consequently, if the manager also cares about short-term considerations ($w < 1$), some inefficient investment in any equilibrium inevitably exists. Our objective is to study and quantify the extent of this inefficiency stemming from managerial myopia. We show below in Proposition 4 that the expected efficiency loss decreases in the manager's long-term incentive, w. In order to provide an upper bound of the efficiency loss, we study the case where $w = 0^+$, i.e., a manager that cares almost exclusively about the short-term price and assigns only an infinitesimal weight to the long-term value. The extreme (an unlikely) case where the manager is fully myopic ($w = 0$) is a special case we will analyze below.

The case of $w = 0^+$ - slight long-term incentives

We next analyze the case where $w = 0^+$, i.e., the case that yields the highest efficiency loss as long as the manager assigns some weight to the long-term value. We begin by providing a rough estimate of the efficiency loss in the case of infinitesimal long-term incentives. This rough estimate will help elucidate some of the insights behind the bound on inefficiency for general distributions, which we later derive in our main result.

This estimate is grounded in the fact that when the manager has even slight long-term preferences, investors cannot hold the belief that she always selects the short-term project, which, as we show below, may only hold in the extreme case of a fully myopic manager.

To understand the driving forces behind our main result note that if $w > 0$, the silence price must reflect the possibility that the manager might pursue the long-term project. Conditional on not disclosing, which guarantees a fixed price in the short term, the manager always chooses the project with the higher value because $w > 0$. As a result, the silence price must reflect the possibility that the manager chose the project with the higher value. Thus the silence price is at least as high as the expected value of project B. Hence, for the uniform distribution we get $P^s > \frac{1}{2} = \mathbb{E}[b]$. The intuition behind this result is simple: under no disclosure, the manager chooses project B unless $b < a$, which is an improvement compared to always selecting B. Formally, $P^s = \mathbb{E}[Max\{a, b\}|a < P^s] > \mathbb{E}[b|a < P^s] = \mathbb{E}[b]$. We show this general result in Proposition 5. Note that no disclosure is always part of the equilibrium since for values of a that are sufficiently close to 0 and values of b that are sufficiently close to 1, even if the silence price were zero, the manager would prefer to choose b.

Next, let's consider the manager's payoff at $t = 1$. Since $w = 0^+$, she almost exclusively cares about the short-term price (at $t = 1$), and by staying silent, she guarantees herself P^s. Therefore, we can describe the manager's strategy as follows:

- If $a > P^s$ select project A and disclose its value.
- If $a \leq P^s$ select the project with the larger value and remain silent.

Combining the result that $P^s > \frac{1}{2}$ with the result that for $a \leq P^s$ the manager always selects the project with the larger value, substantially limits the efficiency loss in equilibrium. In particular, inefficiency can arise only when the value of both projects exceeds the silence price, which is greater than $\frac{1}{2}$.

Lemma 2. *Inefficiency occurs only when* $b > a \geq P^s > \frac{1}{2}$.

From the discussion above, we can conclude that in the case of the uniform distributions:

- Because $P^s > \frac{1}{2}$, inefficiency occurs with a probability of less than 25% (only when $b > a \geq \frac{1}{2}$).
- Even when inefficient investment occurs, the magnitude of the inefficiency is limited as it may occur only for $a > \frac{1}{2}$.

An immediate (though not tight) upper bound of the efficiency loss can be derived by noting that the efficiency loss conditional on $a, b \geq P^s$ is bounded by:

$$\mathbb{E}[Max\{a, b\} - a \mid a, b > \frac{1}{2}] = (\frac{1}{2} + \frac{1}{2} \cdot \frac{2}{3}) - \frac{3}{4} = \frac{1}{12}$$

The efficiency loss is then bounded by:

$$Pr(a > \frac{1}{2}, b > \frac{1}{2}) \cdot \mathbb{E}[Max\{a,b\} - a \mid a,b > \frac{1}{2}] = \frac{1}{4} \cdot \frac{1}{12} = \frac{1}{48}$$

To quantify the effect of managerial myopia on investment efficiency, we calculate the expected efficiency loss as a fraction of full efficiency. Thus, for the case of $w = 0^+$, the efficiency loss relative to the first-best is bounded by

$$\frac{1/48}{2/3} = \frac{1}{32} = 3.125\%$$

Limiting equilibrium ($w \to 0^+$)

We continue this section by explicitly calculating the equilibrium silence price and equilibrium inefficiency for the limit case of $w \to 0^+$.

Let p denote an exogenously given silence price. The manager chooses and discloses a if and only if a is above p. When a is below p, the manager chooses to remain silent and choose the higher-value project. Thus, when both a and b are below p, the expected value of the chosen project is $\mathbb{E}[Max\{a, b|a < p, b < p\}] = \frac{2}{3}p$. On the other hand, if $p < b$ and the manager remains silent, it must be that project B was selected (because if a were greater than p and the manager would have chosen it, its value must have been disclosed). In this case, the expected value of the project is the expectation of b given it is greater than p, which is $\frac{1+p}{2}$.

Putting these calculations together, when the manager optimally responds to silence price p and chooses to remain silent, the expected value of the project is:

$$\begin{aligned}
\mathbb{E}[v|p, silence] &= Pr(b < p)\mathbb{E}[Max\{A, B\}|a < p, b < p] + Pr(p < b)\mathbb{E}[B|p < b] \\
&= p \cdot \frac{2}{3}p + (1-p)\frac{1+p}{2} = \frac{1}{2} + \frac{1}{6}p^2
\end{aligned}$$

In equilibrium, it holds that the silence price is, on average, correct, hence: $\frac{1}{2} + \frac{1}{6}p^2 = p$. Solving this equation yields that the equilibrium silence price in case of $w = 0^+$ is $P^s = 3 - \sqrt{6}$.

One can calculate the expected efficiency loss and show that it is given by $\frac{(1-P^s)^3}{6}$, which for the equilibrium silence price $P^s = 3 - \sqrt{6}$ equals around 0.0151.[15] The tight bound on the efficiency loss as a fraction of the first best is therefore $\frac{0.0151}{2/3} = 2.27\%$.

Because the limit of equilibria is also an equilibrium, it holds that also when $w = 0$ the efficiency loss can be as low as 2.27%.

Unraveling Equilibrium ($w = 0$)

We complete the analysis of the uniform distribution case by examining the scenario where the manager is fully myopic, i.e., $w = 0$. This extreme (and unlikely) scenario raises the possibility of significant efficiency loss. When the manager cares solely about short-term prices, upon no disclosure she is indifferent between projects A and B regardless of their values. This indifference gives rise to a multiplicity of equilibria. The manager's project choice when she does not disclose determines the silence price, and her indifference enables multiple silence prices. One can show that there is a continuum of equilibria for the case of $w = 0$. One of these equilibria is the limit case of $w = 0^+$, which was analyzed above. This is the most efficient equilibrium for $w = 0$. The other end of the continuum of the equilibria is the equilibrium in which the manager always chooses project A, which is the least efficient equilibrium. Below we analyze the least efficient equilibrium when $w = 0$.

When the manager cares solely about short-term prices, investors may believe that the manager always pursues project A, a belief that can hold true in equilibrium. Proposition 3 (below) demonstrates that this equilibrium outcome is the least efficient. In this equilibrium, the unraveling result emerges: investors correctly believe that the long-term project is never implemented. Thus, if the manager remains silent, she must observe the lowest possible value of the short-term project, and the silence price will be zero. Consequently, the ex-ante expected value of the firm equals the expected value of project A, which is $\mathbb{E}[a] = \frac{1}{2}$. The efficiency loss is then $\mathbb{E}[Max\{a, b\} - a] = {}^2/_3 - {}^1/_2 = \frac{1}{6}$.

Thus, in the case of $w = 0$, the relative efficiency loss is given by:

$$\frac{1/6}{2/3} = \frac{1}{4} = 25\%.$$

[15] The expected efficiency loss can be calculated as $\mathbb{E}[v|v = Max\{a, b\}] - \mathbb{E}[v|equilibrium]$.

Under the least efficient equilibrium, the efficiency loss in the case of $w = 0$ is significant, i.e., it leads to the destruction of a quarter of the possible surplus. Nevertheless, this extreme outcome can easily be avoided by providing the manager with infinitesimal long-term incentives. These infinitesimal incentives lead to a change in the silence price from zero to $P^s > \frac{1}{2}$, and thus eliminate the most inefficient decisions that occur when $w = 0$. That is, eliminating the cases when the manager selects a despite it being very low and b being high, that were possible under $P^s = 0$ (in the fully myopic case). This, in turn, results in around a 90% decrease in expected inefficiency when considering slight long-run incentives (from 25% when $w = 0$ to 2.27% when $w = 0^+$). Proposition 4 formally shows that among all the possible equilibria when $w = 0$ the most efficient one leads to the same efficiency as the achieved under the equilibrium when $w = 0^+$. Figure 3 below summarizes the efficiency loss results for all possible values of w, demonstrating the limited inefficiency for any $w > 0$ and the qualitative difference between $w > 0$ and $w = 0$.

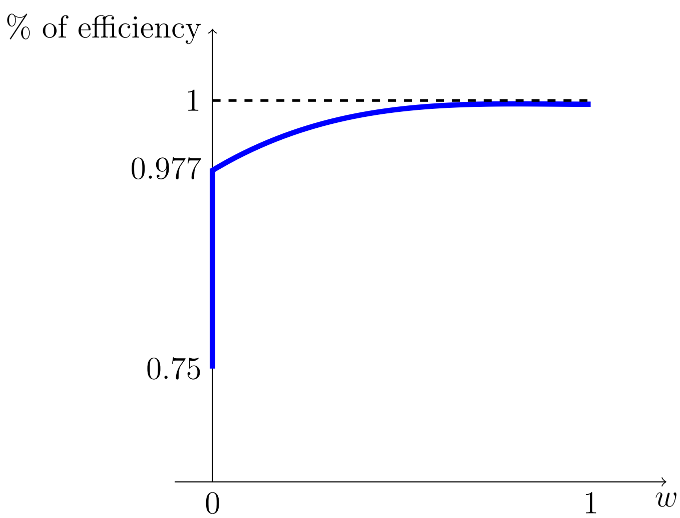

Figure 3: Efficiency as a fraction of the fully efficient choice, for uniform distributions. For $w = 0$ there are multiple equilibria, hence multiple possible (in)efficient outcomes.

4 The General Case: Equilibrium Characteristics

We now study equilibria of the general model with distributions F_A and F_B. We first note that:

Proposition 1. *An equilibrium exists.*

Existence follows from a standard fixed-point argument. For every silence price, we determine the manager's best response and then compute the new silence price it implies. An equilibrium can be found by identifying a fixed point of this mapping (see the Appendix for details).

We next examine how the level of managerial myopia affects the inefficiency of the manager's project selection. As a benchmark, consider the case where the manager always selects efficiently, i.e., pursues A if $a > b$ and B otherwise. We refer to this case as the first-best selection rule, denoting its expected value as:

$$U^{FB} \equiv \mathbb{E}[Max\{A, B\}].$$

Full efficiency is the equilibrium outcome if the manager cares only about the firm's future value (i.e., $w = 1$). However, when the manager also considers short-term prices ($w < 1$), she sometimes selects inefficiently. Specifically, for some values $b > a$, the manager chooses A if it increases her first-period payoff relative to the silence price. This occurs whenever the manager's benefit from boosting the short-term price outweighs her loss from the decrease in long-term value.w Figure 2 that we described above illustrates the efficiency loss. Formally,

Proposition 2. *For any $w < 1$, the first best is not an equilibrium outcome.*

Let U^{EQ} denote the expected value of the project chosen in equilibrium. We define the expected deadweight loss in that equilibrium as:

$$DWL \equiv 1 - \frac{U^{EQ}}{U^{FB}}.$$

DWL is measured as a fraction of the expected value under the first best scenario, making our calculations to be scale-independent. Our starting point is that when $w < 1$ we have a positive DWL.

When $w = 0$, there exists an "unraveling equilibrium" in which the manager always selects A and discloses a for any $a > 0$. This managerial strategy results in a silence price of $P^s = 0$. Given $P^s = 0$, the manager has no incentive to deviate and $U^{EQ} = \mathbb{E}[A]$. We call this equilibrium "unraveling" because it is analogous to the equilibria in disclosure games first derived in Grossman (1981) and Milgrom (1981), where the manager discloses all information, even if it is negative. We conclude that:

Proposition 3. *In the case the manager is fully myopic ($w = 0$), there exists an "unraveling equilibrium" in which the manager always selects and discloses project A, the silence price is zero, and $DWL = 1 - \frac{\mathbb{E}[A]}{\mathbb{E}[\max\{A,B\}]}$.*

Under the least efficient equilibrium, the inefficiency can be substantial. In the example of uniform distributions, we have shown that this may result in $DWL = 25\%$. The main result of this paper is that despite the existence of this equilibrium and the fact that all equilibria are inefficient, when $w > 0$, the expected inefficiency of project selection in equilibrium is much smaller than what the unraveling equilibrium would suggest.

The unraveling equilibrium does not exist when $w > 0$, and when $w = 0$, it is not a unique equilibrium. To understand why multiple equilibria exist when $w = 0$, note that when the silence price is strictly positive and a is below the silence price, a fully myopic manager is indifferent between selecting A or B. This indifference can sustain a range of silence prices in equilibrium. However, this indifference disappears when $w > 0$, establishing a lower bound on the equilibrium silence price. The equilibrium correspondence is upper hemi-continuous, and any limit of equilibria as $w \to 0$ is still an equilibrium for $w = 0$. We will show that any limit of this perturbation corresponds to a much more efficient equilibrium.

Our first main finding is that efficiency increases in w, provided we focus on the least efficient equilibrium. The second part of the theorem states that under certain conditions the set of equilibria converges as w goes to zero to the most efficient equilibrium in the set that corresponds to $w = 0$.

Proposition 4. *1. In the least-efficient equilibrium for every w, the equilibrium silence price and efficiency are increasing in w.*

2. Let F_A and F_B be log-concave distributions, then as the manager becomes more *myopic, $w \to 0$, the equilibrium set converges to the most efficient equilibrium in the case of $w = 0$.*

The class of log-concave distributions includes the uniform distribution and many others (see Bergstrom and Bagnoli (2005)). In the following section, we quantify the efficiency gap when considering $w > 0$ or the limiting equilibrium as $w \to 0$. This analysis is independent of whether there is a unique limiting equilibrium. Hence, our conclusion extends to all distributions and not just those that are log-concave.

Quantifying the Inefficiency Gap

When the manager assigns any positive weight to the future value ($w > 0$), if the value of A falls below the silence price, the manager remains silent at $t = 1$ and selects efficiently. As we argued in the uniform distribution example, this selection ensures that the equilibrium silence price is above the average value of B, thereby limiting the potential inefficiency of the manager's selection. We use this observation to derive a tight bound on the equilibrium inefficiency across all equilibria and distributions.

Theorem 1. *For every $w \in (0, 1)$ and every equilibrium, the DWL is bounded from above by $DWL \leq 1 - \frac{\mathbb{E}[\max\{A, \mathbb{E}[B]\}}{\mathbb{E}[\max\{A,B\}]}$ and positive.*

We have two comments:

- This bound corresponds to the $1 - \frac{31}{48} = 3.125\%$ bound in the uniform example.
- The bound is based on $U^{EQ} \geq \mathbb{E}[\max\{A, \mathbb{E}[B]\}$. Our proof establishes an even tighter bound. In any equilibrium (for $w > 0$)

$$U^{EQ} \geq \int_0^{\mathbb{E}[B]} \int_0^1 \max\{a, b\} dF_B(b) dF_A(a) + \int_{\mathbb{E}[B]}^1 a dF_A(a).$$

One way to interpret Theorem 1 is that there is a significant gap in efficiency between the case of the unraveling equilibrium and any limit of equilibria as $w \to 0$. This gap exists even when we focus on the least efficient equilibrium that corresponds to such a limit.

The proof of the Theorem follows from two observations. First, our argument in Section 3 regarding the silence price being above the unconditional mean of B does not rely on the distribution being uniform. Therefore, we can immediately generalize this result:

Proposition 5. *For any $w > 0$ and any equilibrium, the silence price satisfies $P^s > \mathbb{E}[B]$.*

Second, the equilibrium efficiency increases in the silence price:

Lemma 3. *The equilibrium efficiency of the manager's project choice and the manager's payoff increase with the equilibrium silence price, P^s.*

The intuition behind this lemma is that, from the manager's perspective, the silence price functions an option. Her payoff at $t = 1$ is P^s whether she implements a long-term or a short-term project that she chooses not to disclose. Therefore, her expected payoff increases as P^s increases. Since, by rational expectations, the first-period price in any equilibrium is, on average, equal to the expected value of the selected project, this implies that equilibrium efficiency also increases with P^s.

Combining Lemma 3 with Proposition 5 implies that when moving from the least efficient "unraveling equilibrium" in the case of $w = 0$ to any equilibrium for $w > 0$ (where $P^s > \mathbb{E}[B]$), there will be a discrete improvement in efficiency.

The effect can be decomposed into two parts: First, when the manager remains silent, investors can no longer assume that the project value is the lowest possible, as they would in the unraveling equilibrium. To see this, note that when $w > 0$, if the manager observes a sufficiently low value of A and a high value of B, she chooses B, which is also an efficient choice. Since this event occurs with positive probability, it implies that, in equilibrium, the silence price must be greater than zero. Second, whenever the manager observes a value of A below the silence price, she remains silent regardless of the project selected, and in this case, she selects efficiently to maximize the second period's payoff. This behavior improves market beliefs about the efficiency of the project selection when the manager remains silent, further increasing the silence price. These two forces reinforce each other, resulting in a discrete increase in the silence price and an improvement in efficiency over the unraveling equilibrium.'

As discussed earlier, efficiency loss occurs when the gain from short-term disclosure exceeds the long-run loss, i.e., $\frac{a-P^s}{b-P^s} > w$ (presented by the dotted area on the top-right corner of Figure 2). Inefficiency arises when $a > P^s$, $b > a$, and the manager still selects A due to her short-term incentives. For a given silence price, an increase in w enhances the manager's relative benefit from choosing project B over project A when $b > a$. This shift leads to an increase in the silence price, further amplifying the improvement in efficiency.

Our second main finding is that, due to the economic forces that increase efficiency when $w > 0$, the inefficiency never exceeds 50% of the first-best when the manager cares even slightly about the future, i.e., when $w > 0$. This 50% bound holds across all distributions and all equilibria: even in the worst equilibrium (in cases where multiple equilibria exist) and even in the worst-case scenario across all distributions.

Theorem 2. *For every $w > 0$, every F_A, F_B, and every equilibrium, the DWL is at most* 50%.

While 50% may seem high, it stands in stark contrast to the potential inefficiency of the "unraveling equilibrium". If $\mathbb{E}[B] > 0$ and $\mathbb{E}[A] \approx 0$, then the DWL is approximately 100% in that equilibrium, resulting in the complete disappearance of all surplus. Another implication of this result is that the increase in efficiency between the unraveling equilibrium and a limiting equilibrium is more significant than the increase from a limiting equilibrium when $w = 0$ to the efficient equilibrium corresponding to $w = 1$.

The full equilibrium analysis when $w > 0$ is complex for arbitrary distributions. However, by focusing on the distributions that maximize efficiency loss. According to proposition 4, the worst-case scenario occurs as w converges to zero. In this limit, the manager selects A whenever a is above the silence price, and inefficiency arises only when a is above the silence price and and $b > a$.

While the proof of the theorem is complex, we present the core idea here. We aim to identify the distributions that maximize the relative efficiency loss (DWL), taking into account the impact of any distributional change on the first-best efficiency and the equilibrium price. The proof for the theorem follows from two key lemmas. The first lemma identifies the distribution of the long-term project B, denoted as F_B, that maximizes DWL given:

1. The distribution F_A of the short-term project A.

2. The silence price P^s.

3. The first-best expected efficiency U^{FB}.

We assert that:

Lemma 4. *Given* (1) − (3) *above, the distribution of B that maximizes the DWL assigns positive probability to at most three discrete values, denoted as $B = \{b_0, b_1, b_2\}$.*

The intuition behind the proof of Lemma 4 is that, given F_A, maximizing DWL can be expressed as maximizing a linear function of F_B subject to two linear constraints in F_B as given by conditions (2) and (3) above. Based on a result by Szapiel (1975)[16], the extreme points of the intersection of these two constraints (hyperplanes) with a convex set C can be represented as a convex combination of only three extreme points in C.

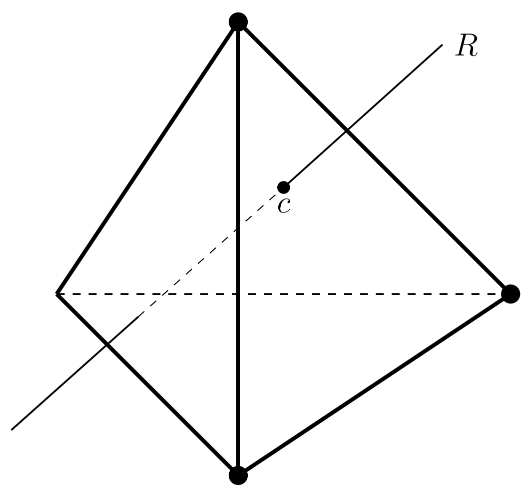

Figure 4: Intersection of two hyperplanes, denoted by R. The dashed line segment represents the intersection with the convex pyramid. Point c is an extreme point within this set and can be expressed as a convex combination of the three marked extreme points of the pyramid.

[16]A more general result can be found in Winkler (1988).

For the second lemma, we take as given the long-term project's support $B = \{b_0, b_1, b_2\}$ as established by Lemma 4, and characterize the support of the short-term project, A which maximize the inefficiency. We argue as follows:

Lemma 5. *Without loss of generality, we can restrict our attention to distributions of the short-term project that assign positive probabilities to at most four values,* $A = \{a_0, a_1, a_2, a_3\}$.

The intuition behind this assertion lies in the fact that efficiency is solely determined by the conditional expected value among the values of both the short and long-term projects. In other words, when the long-term project consists of three possible states, $B = \{b_0, b_1, b_2\}$, along with the silence price that must satisfy $b_0 \leq P^s \leq b_2$, it divides the values of the short-term project into four relevant segments. Each of these segments can be substituted with a corresponding conditional expectation of the project's value based on the equilibrium choice without changing the equilibrium or first-best expected payoffs.

The proof of Theorem 2 is based on finding the weights for $B = \{b_0, b_1, b_2\}$ and $A = \{a_0, a_1, a_2, a_3\}$ that maximize DWL. We then show that this maximum value does not exceed $\frac{1}{2}$.

We find that the distributions leading to the worst-case scenario are actually a 2-point distribution for A and a 2-point distribution for B. In particular, suppose that $A \in \{0, P^+\}$ and $B \in \{0, 1\}$, where P^+ represents a realization just above the silence price (which will be determined in equilibrium, and then we will take the limit).

Denote the probabilities by $\Pr[A = 0] = \pi_0$, $\Pr[A = P^+] = \pi_1$, $\Pr[B = 0] = q_0$, and $\Pr[B = 1] = q_1$.

Note that in this example, our bound on the silence price is tight: the silence price is simply $P^s = q_1$ because the agent remains silent only when $a = 0$, and conditional on that event, the expected value of the maximum of A and B is simply the expected value of B. Moreover, because the manager either receives P^s when she remains silent or gets P^+ when she selects A and discloses, the (limit) equilibrium payoff of the manager is

$$U^{EQ} = P^s = q_1.$$

Finally, the first-best in this case is:

$$U^{FB} = \pi_0 q_1 + \pi_1(q_0 P^s + q_1) = q_1 + \pi_1 q_0 q_1.$$

Taking the ratio, we get:

$$\frac{U^{FB}}{U^{EQ}} = 1 + \pi_1 q_0.$$

By taking π_1 and q_0 close to one, we observe that the ratio can be arbitrarily close to 2, corresponding to a DWL of 50%.

Symmetric distributions

It is important to note that in the previous section, the distributions F_A and F_B that maximize the efficiency loss are extreme. These distributions require that the expected values of both projects approach almost zero in the limit. As a result, even the first-best expected value is nearly zero. Nonetheless, the expected value in equilibrium converges to 0 faster (twice as fast) than the expected value under the first-best.

The bound of the efficiency loss in the general distribution case (50%) is significant, yet much larger than under uniform distributions (2.27%). We now demonstrate that imposing certain restrictions on the distributions yields bounds closer to those of the uniform distribution case than to the general case. Specifically, we analyze the case where the values of both projects are drawn from the same atomless distribution, $F_A = F_B$, which is symmetric around $\mathbb{E}[a] = \frac{1}{2}$. As we shall see, when the manager is fully myopic ($w = 0$), efficiency loss can be as high as $\frac{1}{3}$; When the manager is slightly non-myopic ($w = 0+$), the inefficiency loss is bounded by 4.587%.

The case of a manager with slight long-term incentives

To analyze the efficiency loss when the manager has infinitesimal long-term considerations, $w = 0^+$, it is useful to separate the cases where the project values are above and below the prior mean. Let:

$$\begin{aligned} m &= \mathbb{E}[Max\{A,B\}|a \leq \tfrac{1}{2}, b \leq \tfrac{1}{2}], \\ M &= \mathbb{E}[Max\{A,B\}|a \geq \tfrac{1}{2}, b \geq \tfrac{1}{2}], \\ N &= \mathbb{E}[A|a \geq \tfrac{1}{2}]. \end{aligned}$$

These terms represent the expected value of the project, conditional on selecting the project with the higher value, when the values are below (m) or above (M) the prior mean, and the expected value conditional on being above the prior mean (N). These terms allow us to provide a simple bound on the efficiency loss.

The first-best efficient project selection can be described as follows: if both project values are above (below) the prior mean, which occurs with a probability of $\frac{1}{4}$, then the expected value when the manager selects efficiently is M (m). If one project is above the prior mean, while the other is below, which has a probability of $\frac{1}{2}$, then the expected value when the manager selects efficiently is N. Thus, the first-best expected value of the project is given by:

$$\mathbb{E}[Max\{A,B\}] = \frac{m + M + 2N}{4}.$$

We can bound the expected project value in equilibrium using the terms above and the result from Proposition 5, which states that in equilibrium, the silence price must be above the prior mean, $\frac{1}{2}$. As a lower bound, we shall assume that the silence price equals $\frac{1}{2}$. Based on this, we argue that:

Lemma 6. *The expected value of the project in equilibrium is at least*

$$\mathbb{E}[v|w = 0^{+}] > \frac{m + 3N}{4}.$$

Thus, a bound for the efficiency loss as a fraction of the first best outcome is given by:

$$1 - \frac{\mathbb{E}[v|equilibrium]}{\mathbb{E}[Max\{A,B\}]} = 1 - \frac{m + 3N}{m + M + 2N}$$

Using the properties of symmetric distributions, we can further simplify this and conclude that:

Lemma 7. *Assuming $w = 0^+$, the efficiency loss relative to the first-best expected value is bounded by* $\frac{M-N}{1+2M}$.

We are now ready to evaluate the maximal relative efficiency loss in the case where the manager has infinitesimal long-term incentives, $w = 0^+$. The bound of relative efficiency loss, $\frac{M-N}{1+2M}$, depends on the distribution F. Differentiation of this expression with respect to M reveals that the efficiency loss increases in M.[17] Hence, an upper bound is obtained when we change the distribution F in a way that keeps N fixed but increases M.[18] Thus, we can take the limit of such changes as an upper bound of the relative efficiency loss.

Proposition 6. *Suppose $F_A = F_B$ is a symmetric distribution. If the manager has infinitesimal long-term considerations, i.e., $w = 0^+$, then the maximum efficiency loss as a fraction of full efficiency is less than* 4.6%.

Proof. As described above, to maximize the relative efficiency loss, one can take the limit of conditional-mean-preserving spreads over the interval $[\frac{1}{2}, 1]$, while keeping N fixed. In this limit, only $\frac{1}{2}$ and 1 are assigned positive probabilities. Let q be the probability assigned to $\frac{1}{2}$ in the limit. Then, we have $N = q \cdot \frac{1}{2} + (1-q) \cdot 1$ and $M = q^2 \cdot \frac{1}{2} + (1-q^2) \cdot 1$.

Therefore, in this case

$$\frac{M-N}{1+2M} = \frac{q^2\frac{1}{2} + (1-q^2) - q\frac{1}{2} - (1-q)}{1 + 2(q^2\frac{1}{2} + (1-q^2))} = \frac{1}{2}\frac{q(1-q)}{3-q^2}.$$

This last expression is maximized at $q = 3 - \sqrt{6}$. Substituting $q = 3 - \sqrt{6}$, we get $\frac{M-N}{1+2M} = \frac{1}{2} \cdot \frac{q(1-q)}{3-q^2} = 0.04587$. □

The case of a fully myopic manager

We conclude our analysis with the case of a fully myopic manager with projects of symmetric distributions. As we explained above, when $w = 0$, in the least efficient

[17] Note that $\frac{\partial}{\partial M}\left(\frac{M-N}{1+2M}\right) = \frac{1+2M-2(M-N)}{(1+2M)^2} = \frac{1+2N}{(1+2M)^2} > 0$.

[18] This change is to alter the distribution to the right of the prior mean in a conditional-mean-preserving spread around the conditional mean N and make symmetric changes to the distribution to the left of the prior mean.

equilibrium, the manager always selects A. The expected value of the project in this case is $\mathbb{E}[A] = \frac{1}{2}$.

The first-best efficiency is $\mathbb{E}[Max\{A, B\}]$, which depends on the specific distribution of the projects. The maximal efficiency loss is determined by the (symmetric) distribution that maximizes inefficiency. We argue that this occurs with the binomial distribution $Pr(0) = Pr(1) = \frac{1}{2}$. To see why, consider mean preserving spreads to A, B given by $A_{\epsilon_A} = A + \epsilon_A, B_{\epsilon_B} = B + \epsilon_A$ where $\mathbb{E}(\epsilon_A|A) = \mathbb{E}(\epsilon_B|B) = \frac{1}{2}$. We argue that

Lemma 8. $\mathbb{E}[Max\{A, B\}] \leq \mathbb{E}[Max\{A_{\epsilon_A}, B_{\epsilon_B}\}]$

Taking the maximal mean-preserving spread leads to a binary distribution $Pr(0) = Pr(1) = \frac{1}{2}$. In that case $\mathbb{E}[Max\{A, B\}] = \frac{3}{4}$. We conclude that:

Corollary 1. *If the manager is fully myopic, $w = 0$, then with symmetric distributions, the maximum efficiency loss, as a fraction of full efficiency, is* $\frac{3/4 - 1/2}{3/4} = \frac{1}{3}$.

5 Discussion

We investigated the impact of managerial short-termism on efficiency based on a strategic voluntary disclosure model. Our analysis reveals that while short-termism leads to investment inefficiencies, the extent of those inefficiencies is relatively limited. Notably, even marginal consideration for long-term value by managers significantly mitigates investment inefficiency. This reduction stems from a decrease in investors' pessimism regarding the manager's investment choice when information is not disclosed, leading to efficient investment in cases where inefficiency was most severe.

Interestingly, while the market efficiently prices the firm based on available information, inefficient investment decisions can occur only when the manager discloses information and prices perfectly reflect the firm's value. When the manager remains silent, prices reflect the expected value, and the manager invests efficiently as the pressure to provide results in the short term is muted. This underscores the nuanced interplay between managerial actions, market dynamics, and investment efficiency, emphasizing the importance of addressing short-termism within a broader framework of corporate governance and policy interventions.

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6 Appendix

Proof of Lemma 1:

Proof. Given a silence price p, the manager reveals a if $a > p$ and chooses A if $a > b$ or $a < b$ and $(1-w)\,p + wb < a$. For any given $w \in (0,1)$ and p, calculating the expected value of the project given no disclosure by the manager yields (see additional details on this calculation below)

$$H\left(p|w\right) = \frac{w\left(p^3 - 3p + 2\right) + p^3 + 3p}{3\left(w\left(1-p\right)^2 + 2p\right)}.$$

As we discussed in Section 2, the equilibrium silence price is given by the fixed point

$$P^S = H\left(P^S|w\right),$$

which in this case becomes:

$$p = \frac{w\left(p^3 - 3p + 2\right) + p^3 + 3p}{3\left(w\left(1-p\right)^2 + 2p\right)}$$

Multiplying it out and collecting terms with w we get that this equation is equivalent to:

$$w = \frac{p\left(p^2 - 6p + 3\right)}{2\left(p-1\right)^3} \equiv \gamma\left(p\right)$$

Note that the silence price has to be less than $\mathbb{E}\left[\max\left\{A, B\right\}\right]$ (since disclosure removes only types that are higher than the silence price and $\mathbb{E}\left[\max\left\{A, B\right\}\right]$ would be the price if no type disclosed). Moreover, for any $p \leq 0.5$ it is easy to verify that $\gamma\left(p\right) \leq 0$, so the only relevant range is between $\mathbb{E}\left[B\right]$ and $\mathbb{E}\left[\max\left\{A, B\right\}\right]$, which for the uniform distribution case is between $\frac{1}{2}$ and $\frac{2}{3}$.[19]

To verify algebraically that $\gamma\left(p\right)$ is increasing in the range $\left(\frac{1}{2}, \frac{2}{3}\right)$ note that:

$$\gamma'\left(p\right) = \frac{3\left(\left(p+1\right)^2 - 2\right)}{2\left(1-p\right)^4}$$

[19] As we showed in Proposition 5, for any $w \in (0,1)$ the silence price is above $\mathbb{E}\left[B\right]$.

which is strictly positive in this range.

This allows us also to see that as w increases in the range $(0, 1)$, the unique equilibrium silence price increases continuously from $P^s (w = 0^+) = 3 - \sqrt{6} = 0.550\,51$ to $P^s (w = 1^-) = 0.596\,07$.

To derive the $H(p|w)$ calculation, we take the following steps:

The manager reveals a if $a > p$ and chooses A if $a > b$ or $a < b$ and $(1-w)p + wb < a$. Hence disclosure happens with probability

$$\Pr\left(disclosure|P^S = p\right) = \frac{1}{2} - \frac{p^2}{2} + \frac{1}{2}(1-p)^2(1-w)$$

Accordingly, the probability of the manager not disclosing A is

$$\begin{aligned}\Pr(ND|P^s = p) &= \frac{1}{2} + \frac{p^2}{2} - \frac{1}{2}(1-p)^2(1-w) \\ &= \frac{1}{2}\left(w(1-p)^2 + 2p\right)\end{aligned}$$

Conditional on no disclosure the expected project's value is

$$\begin{aligned}H(p|w) &= \frac{p^2 \frac{2}{3}p + p(1-p)\frac{1+p}{2} + \int_p^{(1-w)p+w}\left(\int_{\frac{a-(1-w)p}{w}}^1 bdb\right)da}{\frac{1}{2}\left(w(1-p)^2 + 2p\right)} \\ &= \frac{w(p^3 - 3p + 2) + p^3 + 3p}{3\left(w(1-p)^2 + 2p\right)}\end{aligned}$$

as claimed above. The first term in the numerator corresponds to the case where A and B are below p. The second term is the case where a is below p and b is above. The last term corresponds to the case where a is above p but b is sufficiently high so that $(1-w)p + wb > a$. □

Proof of Lemma 2:

Proof. As evident from the above, the manager may select an inefficient project only when $a \geq P^s$. Note that when $b \leq P^s$ the manager selects the efficient project. This

holds true because when $a \geq P^s$, the manager selects A, but this selection is efficient since $b < a$.
Furthermore, when $a > b \geq P^s$ the manager efficiently chooses A because choosing B yields a payoff of P^s at $t = 1$. □

Proof of Proposition 1:

Proof. The equilibrium existence follows from the intermediate value theorem. For an specified silence price p, let $H(p|w) \equiv \mathbb{E}\left[\max\left\{A, B\right\}|A \leq p \text{ or } p < A \leq (1-w)\,p + wB\right]$

Then, an equilibrium silence price is any fixed point:

$$P^s = H\left(P^s|w\right).$$

We note that:

- $H(p|w)$ is continuous in p.
- $H(0|w) \geq 0$: This follows as when the silence price is zero then the manager always discloses a if she selects the short-term project. In this case, the long-term project is selected only if $(1-w)\cdot 0 + w \cdot b > 0$, which for $w \neq 0$ is equivalent to $b > \frac{a}{w}$. Hence, in this case, the expected value conditional on silence is

$$H(0|w) = \mathbb{E}[b|b > \frac{a}{w}] > \mathbb{E}[b] > 0.$$

  For $w = 0$, we get that $H(0|w) = 0$.
- $H(1|w) < 1$: This follows as the manager always remains silent. In this case,

$$H(1|w) = \mathbb{E}[max\{A, B\}] < 1.$$

□

Proof of Proposition 2:

Proof. We first note that in any equilibrium $P^s < 1$, as otherwise, the manager would always remain silent, which would imply that the average value is strictly lower than 1. That would contradict $P^s = 1$.

The claim then follows as $\exists a, b$ (a set with a positive measure) such that $P^s < a < b$ and the manager benefits from selecting A and disclosing it. In particular, suppose that $b = 1$. Then, for $w < 1$, the payoff from selecting project B is $(1-w) \cdot P^s + w \cdot 1 < 1$. If, instead, she selects A and discloses it, she receives a. The claim follows as for short-term projects for which $a \in ((1-w) \cdot P^s + w \cdot b, 1)$ she would prefer it over the long-term project, which is inefficient. The same argument holds for b slightly below 1, implying a positive probability of an inefficient outcome.

□

Proof of Proposition 3:

Proof.

Note that the following is an equilibrium. The manager always selects the short-term project A and discloses its realization a. The silence price, in this case, is $P^s = 0$. The logic behind this construction is the unraveling principle that was first derived in Grossman (1981) and Milgrom (1981). The market holds the worst belief in the case of silence, which is off the equilibrium path. Because the manager always selects project A the expected value is $\mathbb{E}[A]$, thus $DWL = 1 - \frac{\mathbb{E}[A]}{\mathbb{E}[\max\{A,B\}]}$. □

Proof of Proposition 4:

Proof. 1. Recall that inefficiency occurs when $P^s < a < b$ and $(1-w)P^s + wb < a$. Let $P_1^s < P_2^s$. Note that for any (a, b) such that the manager chooses inefficiently under P_2^s, she also chooses inefficiently under P_1^s, because $P_2^s < a < b$ and $(1-w)P_2^s + wb < a$ implies that also $P_1^s < a < b$ and $(1-w)P_1^s + wb < a$. However, there exist (a, b) such that $P_1^s < a < P_2^s$ where the manager chooses inefficiently under P_1^s but chooses efficiently under P_2^s, because for any $a < P_2^s$ the manager remains silent and chooses efficiently. Thus, efficiency increases in P^s, and so, also the manager's utility increases in P^s.

Let $H(p, w)$ denote the average "quiet" type when the silence price is p and given the weight in the manager's utility function is w. Suppose that $w_2 > w_1$ and consider the lowest-equilibrium price, P_2^s, when $w = w_2$. That is the lowest P_2^s such that $H(P_2^s, w_2) = P_2^s$. We shall argue that $H(P_2^s, w_1) < P_2^s$.

To see why this is true, note that when the manager cares more about the short-term price, i.e., with a lower $w_1 < w_2$ but the same silence price P_2^s, the manager changes her project selection. She chooses A for some $a > P_2^s$ and $b > a$. This removes from the set of types who remain silent values of $b > P_2^s$, which will result in a lower average $H(P_2^s, w_1)$.

Since $H(0, w_1) > 0$ we conclude that there exists $P_1^s < P_2^s$ for which $H(P_1^s, w_1) = P_1^s$. Hence, for every equilibrium with $w = w_2$ there exists an equilibrium for $w = w_1$ with a lower silence price. Since efficiency is monotonically increasing in the silence price, the claim follows.

2. The equilibrium for $w = 0^+$ is such that the manager chooses according to $Max\{a, b\}$ if $a < P^s$ and chooses A if $P^s < a$. Thus, the equilibrium is a fixed point such that

$$P^s = \mathbb{E}[Max a, b | a < P^s]$$

. When $w = 0$ there are multiple equilibria. The most efficient equilibrium is such that the manager chooses $Max\{a, b\}$ if $a < P^s$, where P^s is the highest possible. Thus, for $w = 0^+$ there is a unique equilibrium that converges to the most efficient equilibrium under $w = 0$ if there is a unique solution to $P^s = \mathbb{E}[Max\{a, b\}|a < P^s]$. Let F_A and F_B be log-concave distributions, then $Max\{a, b\} \sim H(x)$ is also log-concave because $H(x) = F_A(x)F_B(x)$ and the product of two log-concave distributions is also log-concave. Now,

$$\frac{\partial \mathbb{E}[Max\{a, b\}|a < P^s]}{\partial P^s} \leq \frac{\partial \mathbb{E}[Max\{a, b\}|a < P^s, b < P^s]}{\partial P^s} < 1$$

The first inequality holds because for $P^s < b$ the effect is negligible. The second inequality holds because $H(x)$ is log-concave, see Lemma 1 in Bergstrom and Bagnoli (2005).[20] Since for $P^s = 0$ we get that $\mathbb{E}[Max\{a, b\}|a < P^s] = \mathbb{E}[b]$ and for $P^s = 1$ we get that $\mathbb{E}[Max\{a, b\}|a < P^s] = \mathbb{E}[Max\{a, b\}]$, where $\mathbb{E}[b] < \mathbb{E}[Max\{a, b\}] < 1$. Hence, there is a unique solution to the equation above.

□

Proof of Theorem 1:

[20] This result is based on Goldberger (1983).

Proof. Proposition 5 argues that $P^s > \mathbb{E}[B]$. When $w > 0$, in equilibrium, if $P^s < a$ then the manager selects A unless $a < b$ and $a < (1-w)P^s + wb$. Thus, for $P^s < a$ the expected value is at least $\mathbb{E}[A|P^s < a]$. Also, because $P^s > \mathbb{E}[B]$, we get that for $a < P^s$, the expected value of the project is at least $\mathbb{E}[B]$. Taking together, we get that the equilibrium expected value of the chosen project is at least $\mathbb{E}[max\{A, \mathbb{E}[B]\}]$.

For the tighter bound, note that when $w > 0$, if the manager remains silent, she chooses efficiently, i.e., she chooses $max\{a, b\}|a < P^s$. Because $\mathbb{E}[B] < P^s$, similar to the argument above, we get that the equilibrium expected value is at least

$$\int_0^{\mathbb{E}[B]} \int_0^1 \max\{a, b\} dF_B(b) dF_A(a) + \int_{\mathbb{E}[B]}^1 a dF_A(a).$$

For the proof that all equilibria are inefficient for any $w < 1$, see Proposition 2. □

Proof of Proposition 5:

Proof. Note that in case $w = 0+$, the belief in case of silence is that it is either project B or project A with a realization $a < P^s$; however, in this case, we also have that $a > b$. That is, $P^s = \mathbb{E}[Max\{a, b\}|a < P^s] > \mathbb{E}[b|a < P^s] = \mathbb{E}[b]$. Hence, the belief in the case of silence is better than the prior belief regarding B. □

Before proving Theorem 2, we prove Lemmas 4 and 5:

Proof of Lemma 4:

Proof. Given distributions F_A and F_B the first-best surplus is

$$U^{FB} = \int h_1(b)\, dF_B(b)$$

where

$$h_1(b) = F_A(b)\, b + \int_b^1 a dF_A(a)\, .$$

The equilibrium silence price satisfies

$$P^s = \int h_2(b)\, dF_B(b)$$

where

$$h_2(b) = \begin{cases} \frac{1}{F_A(P)} \left(F_A(b)\, b + \int_b^P a dF_A(a) \right) & \text{for } b \leq P^s \\ b & \text{for } b > P^s \end{cases}$$

Finally, the DWL can be written as

$$\int h_3(b)\, dF_B(b)$$

where

$$h_3(b) = \begin{cases} 0 & \text{for } b \le P \\ (F_A(b) - F_A(P))\, b - \int_P^b a dF_A(a) & \text{for } b > P \end{cases}$$

Maximizing DWL w.r.t. to F_B is:

$$\begin{aligned} & \max_{F_B} \int h_3(b)\, dF_B(b) \\ & s.t. \\ U^{FB} &= \int h_1(b)\, dF_B(b) \\ P &= \int h_2(b)\, dF_B(b) \end{aligned}$$

where the three functions h_1, h_2, h_3 are bounded and continuous.

By Szapiel (1975) and Proposition 2.1 in Winkler (1988), if a solution to this problem exists (i.e., if the set of B that satisfy the constraints is non-empty) then a solution exists such that F_B assigns positive probability to at most 3 points: $B \in \{b_0, b_1, b_2\}$. □

Proof of Lemma 5:

Proof. Following Lemma 4 we consider $B \in \{b_0, b_1, b_2\}$, and denote by q_i the probability assigned to b_i.

First, note that we must have $P \ge b_0$ in any equilibrium. Otherwise, the manager would never choose A and remain silent. Selecting A and remaining silent implies that $A < P < B$. It is dominated by selecting B as it yields the same current payoff and a higher future one. Also, note that to maximize DWL, we can without loss of generality consider the case of $P < b_2$ because if $P \ge b_2$ the equilibrium is efficient. So, we consider two remaining cases.

Case 1: $b_0 \le P \le b_1 \le b_2$.

- Consider now any distribution of A below P. If we replace that distribution with a degenerate distribution with value $a_0 \equiv \mathbb{E}[\max\{b_0, A | A < P\}]$ (and keep the

rest of the distributions unchanged), then the expected equilibrium price and surplus, as well as first-best surplus do not change.
Reason: when A is in that range, it is never disclosed. Conditioning on $A < P$, the choice in the equilibrium and in the first best is the same, that is $\max\{A, B\}$. When $B = b_i$ for $i \in \{1, 2\}$, the value of $(\max\{A, B\} | B = b_i, A < P) = b_i$ and that is unchanged when we make this replacement of the distribution of A. When $B = b_0$, the average value $\mathbb{E}[\max\{A, B\} | B = b_0, A < P] = a_0$ and hence is the same as when we replace the distribution.

- Similarly, consider the range of A between $[P, b_1]$, (b_1, b_2) and above b_2. Let the expected values be $a_1 = \mathbb{E}[A|A \in [P, b_1]]$, $a_2 = \mathbb{E}[A|A \in (b_1, b_2)]$, and $a_3 = \mathbb{E}[A|A > b_2]$. Then, if we replace the distribution of A with the distribution that assigns probability $\pi_1 = \Pr(A \in [P, b_1])$ to a_1, $\pi_2 = \Pr(A \in (b_1, b_2))$ to a_2 and $\pi_3 = \Pr(A \geq b_2)$ to a_3, the U^{FB}, P and U^{eq} do not change. The reason is that these changes affect only distribution of A above P, so that has no impact on the computation of P. Moreover, for U^{FB} and U^{eq} we can condition first one each of the events of B taking one of the values and A falling in one of these ranges, to verify that the replacement leaves all these conditional values and probabilities of these events unchanged.

- That leaves us with 4 possible values of A, as claimed in Step 2, and the possible configuration is

$$b_0 \leq a_0 \leq P \leq a_1 \leq b_1 \leq a_2 \leq b_2 \leq a_3.$$

Case 2: $b_0 \leq b_1 \leq P \leq b_2$

- Consider realizations of A and B below P. If we replace those distributions with a degenerate distributions with values $b_0 = a_0 \equiv \mathbb{E}[\max\{B, A | A < P, B < P\}]$ (and keep the rest of the distributions unchanged), then the expected equilibrium surplus and first-best surplus do not change.
Reason: Consider first P. We can write it as

$$P = q_2 b_2 + (1 - q_3)\,\mathbb{E}[\max\{A, B | A < P, B < P\}]$$

and that is unchanged under the replaced distribution. For U^{FB} we have

$$U^{FB} = q_2\,\mathbb{E}\left[\max\left\{b_2, A\right\}\right] + (1 - q_2)\,\mathbb{E}\left[\max\left\{A, B\right\}|B < P\right]$$

and that is also unchanged under the replaced distribution. Finally, the equilibrium surplus is the same again: when the manager remains silent with the same probability under both distributions and the expected surplus upon silence is P. And the manager discloses A when $A > P$, and we have not changed that distribution.

- Similarly, consider the range of A between $[P, b_2]$, and above b_2. Let the expected values be $a_1 = \mathbb{E}\left[A|A \in [P, b_2]\right]$, and $a_2 = \mathbb{E}\left[A|A > b_2\right]$. Then if we replace the distribution of A by the distribution that assigns probability $\pi_1 = \Pr\left(A \in [P, b_2]\right)$ to a_1, $\pi_2 = \Pr\left(A \geq b_2\right)$ to a_2, the U^{FB}, P and U^{eq} do not change. To see this, these changes affect only the distribution of A above P, so that has no impact on the computation of P. Moreover, for U^{FB} and U^{eq} we can condition first one each of the events of B taking one of the values and A falling in one of these ranges, to verify that the replacement leaves all these conditional values and probabilities of these events unchanged.

That leaves us with 2 possible values of B and 3 possible values of A, as claimed in Step 2, and the possible configuration is (with renaming of the values of B):

$$b_0 \leq a_0 \leq P \leq a_1 \leq b_1 \leq a_2. \tag{1}$$

□

Proof of Theorem 2

Proof. We refer to the different cases in the proof of Lemma 5. We start with the simpler **Case** 2 **derived in Step** 2. There, we have derived that we can, without loss, consider only distributions that satisfy (1) .

Denote by $U_i^{FB} = \mathbb{E}\left[U_i^{FB}|A = a_i\right]$ to be the expected first-best surplus conditional on $A = a_i$. Define the conditional expected equilibrium surplus analogously. Then, since the equilibrium is efficient when $A = a_0$ or $A = a_2$, the relative inefficiency can be written as:

$$DWL = \frac{\pi_1 \left(U_1^{FB} - U_1^{eq}\right)}{\sum \pi_i U_i^{FB}}$$

Note that $U_2^{FB} \geq U_1^{FB}$ (because $U_2^{FB} = a_2$ and $U_1^{FB} = \mathbb{E}[\max\{a_1, B\} \leq b_1]$). Moreover, note that shifting probability from π_2 to π_1 does not change P. Hence, by moving probability from π_2 to π_1, we make the denominator weakly smaller and the numerator strictly larger. Hence, the worst DWL is attained by a distribution that assigns no probability to a_2.

This implies that to find the worst-case DWL, it is sufficient to consider $2-point$ distributions of $A \in \{a_0, a_1\}$ with $b_0 \leq a_0 < P \leq a_1 \leq b_1$. In this case, the expected first-best surplus is:

$$U^{FB} = \mathbb{E}[\max\{A, B\}] = q_0 \, \mathbb{E}[A] + q_1 b_1$$

and the relative inefficiency is

$$DWL = \frac{\pi_1 q_1 (b_1 - a_1)}{q_0 \, \mathbb{E}[A] + q_1 b_1}. \tag{2}$$

(note that the equilibrium is inefficient only when the realizations are a_1 and b_1).

Expression (2) decreases in a_1, so to find the worst-case scenario we can substitute $a_1 = P$. The equilibrium silence price is:

$$P = q_0 a_0 + q_1 b_1.$$

Therefore we get

$$DWL \leq \frac{\pi_1 q_1 (b_1 - (q_0 a_0 + q_1 b_1))}{q_0 \, \mathbb{E}[A] + q_1 b_1} = \frac{\pi_1 q_1 q_0 (b_1 - a_0)}{q_0 \, \mathbb{E}[A] + q_1 b_1}$$

Next, we see that this expression is decreasing in a_0. So DWL is maximized when $a_0 = b_0$:

$$DWL \leq \frac{\pi_1 q_1 q_0 (b_1 - b_0)}{q_0 (\pi_0 b_0 + \pi_1 (q_0 b_0 + q_1 b_1)) + q_1 b_1}$$

This expression is decreasing in b_0 so we take $b_0 = 0$.

$$DWL \leq \frac{\pi_1 q_1 q_0 b_1}{q_0 \pi_1 q_1 b_1 + q_1 b_1} = \frac{q_0 \pi_1}{q_0 \pi_1 + 1} \leq \frac{1}{2}$$

This expression is increasing in $q_0 \pi_1$ and hence is maximized when $q_0 \pi_1 \to 1$, yielding the last inequality.

We are left with **Case 1 derived in Step 2,** with the possible configuration:

$$b_0 \leq a_0 \leq P \leq a_1 \leq b_1 \leq a_2 \leq b_2 \leq a_3.$$

The same reasoning as above implies that to find the worst-case scenario, we can assume there is no weight on a_3 (because realizations of A above b_2 are chosen efficiently in equilibrium, not contributing to inefficiency).

So we are left with:

$$b_0 \leq a_0 \leq P \leq a_1 \leq b_1 \leq a_2 \leq b_2.$$

The expected first-best surplus is:

$$U^{FB} = q_0 \, \mathbb{E}[A] + q_1 \left((1 - \pi_2) \, b_1 + \pi_2 a_2 \right) + q_2 b_2$$

The expected relative inefficiency is therefore

$$DWL = \frac{\pi_1 \left(q_1 (b_1 - a_1) + q_2 (b_2 - a_1) \right) + \pi_2 q_2 (b_2 - a_2)}{q_0 \, \mathbb{E}[A] + q_1 \left((1 - \pi_2) \, b_1 + \pi_2 a_2 \right) + q_2 b_2}$$

This expression is decreasing in a_1, and a_2, so the worst-case scenario is to set $a_1 = P$ and $a_2 = b_1$. The equilibrium price is

$$P = q_0 a_0 + q_1 b_1 + q_2 b_2.$$

That gives:

$$DWL \leq \frac{\pi_1 \left(q_1 (b_1 - P) + q_2 (b_2 - P) \right) + \pi_2 q_2 (b_2 - b_1)}{q_0 \, \mathbb{E}[A] + q_1 b_1 + q_2 b_2}$$

This expression is decreasing in a_0 (recall that P increases in a_0) and so the worst-case scenario is to set $a_0 = b_0$ and then set both values to 0 :

$$DWL \leq \frac{\pi_1 (q_1 (b_1 - P) + q_2 (b_2 - P)) + \pi_2 q_2 (b_2 - b_1)}{q_0 (\pi_1 P + \pi_2 b_1) + q_1 b_1 + q_2 b_2}.$$

where

$$P = q_1 b_1 + q_2 b_2$$

Simplifying we get

$$DWL \leq \frac{\pi_1 q_0 (q_1 b_1 + q_2 b_2) + \pi_2 q_2 (b_2 - b_1)}{(\pi_1 q_0 + 1)(q_1 b_1 + q_2 b_2) + \pi_2 q_0 b_1}$$

This expression is decreasing in b_1 hence it is maximized when $b_1 = a_1 = P$, which in turn implies

$$P = q_1 P + q_2 b_2 \rightarrow P = b_2 \frac{q_2}{1 - q_1}$$

Substituting this we get

$$\begin{aligned} DWL &\leq \frac{\pi_1 q_0 \left(q_1 b_2 \frac{q_2}{1-q_1} + q_2 b_2 \right) + \pi_2 q_2 \left(b_2 - b_2 \frac{q_2}{1-q_1} \right)}{(\pi_1 q_0 + 1) \left(q_1 b_2 \frac{q_2}{1-q_1} + q_2 b_2 \right) + \pi_2 q_0 b_2 \frac{q_2}{1-q_1}} \\ &= \frac{(\pi_1 + \pi_2) q_0}{(\pi_1 + \pi_2) q_0 + 1}. \end{aligned}$$

This last expression is increasing in $(\pi_1 + \pi_2) q_0$ and hence is maximized as $(\pi_1 + \pi_2) q_0 \rightarrow 1$, yielding again the bound $DWL \leq \frac{1}{2}$.

□

Proof of Lemma 6:

Proof. We shall compute the expected value of the project by considering three events:

1. $A, B < \frac{1}{2}$. The manager remains silent and chooses efficiently. Thus, the expected value in this case equals m.

2. $A < \frac{1}{2}, B \geq \frac{1}{2}$. The manager remains silent and chooses efficiently (B). Thus, the expected value in this case equals N.

3. $A \geq \frac{1}{2}$. The manager may or may not choose efficiently. A lower bound would be to always select A, which would lead to an expected value of N

The first two events occur with probability $\frac{1}{4}$ while the third event occurs with a probability of $\frac{1}{2}$. Hence, we conclude that the expected value is at least

$$\mathbb{E}[v|w = 0^+] > \frac{m + 3N}{4}$$

□

Proof of Lemma 7:

Proof. The first-best expected value is given by $\frac{m+M+2N}{4}$. The expected value in equilibrium is at least $\frac{m+3N}{4}$. Thus, the relative efficiency loss relative to the first best is less than

$$\frac{\frac{m+M+2N}{4} - \frac{m+3N}{4}}{\frac{m+M+2N}{4}} = \frac{m + M + 2N - m - 3N}{m + M + 2N} = \frac{M - N}{1 + 2M}$$

The last equality is based on the fact that $M = m+2N-1$ (or equivalently $m+2N = 1 + M$). To see why this is true, Let $\tilde{M} = \mathbb{E}[Min\{A, B\}|a \geq \frac{1}{2}, b \geq \frac{1}{2}]$. From the symmetry of the distribution, we get that $\tilde{M} = 1 - m$. Also, $\frac{\tilde{M}+M}{2} = N$. Therefore, $M = m + 2N - 1$. □

Proof of Lemma 8:

Proof. Note that $\mathbb{E}(A_{\epsilon_A}) = \mathbb{E}(A) = \mathbb{E}(B_{\epsilon_A}) = \mathbb{E}(B)$. This implies that conditional on the event $A > B$ and $A_{\epsilon_A} > B_{\epsilon_B}$ we have $\mathbb{E}[Max\{A, B\}] = \mathbb{E}[Max\{A_{\epsilon_A}, B_{\epsilon_B}\}]$. Conditional on the event $A > B$ and $A_{\epsilon_A} < B_{\epsilon_B}$ we have $\mathbb{E}[Max\{A, B\}] < \mathbb{E}[Max\{A_{\epsilon_A}, B_{\epsilon_B}\}]$. Hence,

$$\mathbb{E}[Max\{A, B\}|A > B] < \mathbb{E}[Max\{A_{\epsilon_A}, B_{\epsilon_B}\}|A > B]$$

The proof follows as a similar claim holds when we condition on $B > A$. □