Balloon or Bubble:
Where is Grade Inflation Leading Us to?

by

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An honors thesis submitted in partial fulfillment

of the requirements for the degree of

Bachelor of Arts

Business and Economics Honors Program

NYU Shanghai

May 2024

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Abstract

This paper explores the economic implications of grade inflation within the context of higher education, focusing on the signaling mechanisms used by students and universities to navigate the labor market. The analysis delves into the game-theoretical models to illustrate how universities might manipulate grading policies as strategic tools to influence student enrollment and maximize their revenue. The findings reveal that universities, acting as monopolists, have a vested interest in inflating grades to attract a broader range of students while maximizing tuition-based revenues. In competitive academic environments, this paper discusses how different grading policies can lead to a variety of equilibrium scenarios, including pooling equilibria where students' abilities cannot be distinguished by grades alone, and separating equilibria where grades accurately reflect student abilities. The study also examines the impact of these policies on student welfare and the overall efficiency of the educational market.

Keywords: Grade Inflation; Higher Education; Signaling; University Competition; Educational Policy

Preface

The phenomenon of grade inflation among students in private universities in the United States has ceased to be a novelty. Since the 1960s, the average grades have steadily risen from below 3.0 to recent levels nearing their ceiling. This topic is uncontroversial, yet as a current university student, the impact of grade inflation on the higher education system is of particular relevance to me.

The motivation for this research originated from my experience studying human capital in the Labor Economics course taught by Professor Katarína Borovičková at NYU. My deeper understanding of signaling models was further developed in a course on Advanced Microeconomic Theory taught by Professor Boyan Jovanovic at NYU. This ultimately led me to formulate the research question addressed in this study.

Under the guidance and with the support of Professor Jovanovic, I conducted an extensive theoretical exploration of grade inflation in American private universities, integrating my economic intuition with the knowledge acquired during my studies. This research has brought me closer to comprehending the mechanisms behind grade inflation and discerning its potential directions and implications for the university education system, enabling a more informed perspective on the meaning of grades as an indicator.

Acknowledgements

First and foremost, I would like to express my deepest gratitude to my thesis advisor, Professor Boyan Jovanovic, for his substantial support and insightful guidance. Whether it was steering the direction of my research or highlighting critical issues, you have always been a spark for significant breakthroughs in my study.

I am also immensely grateful to Professor Kyle Chauvin for his patience and the invaluable advice he provided throughout my research journey. Presenting my derivations in your office has become a cherished memory of my senior year. Thank you for patiently listening to each of my nascent ideas.

Additionally, I extend my thanks to my second reader, Professor Weiwei Weng, for her early discussions about the feasibility of empirical research. Although this aspect did not materialize in the final work, I am thankful for your insights that opened up another potential avenue for my research.

Moreover, I am thankful to Professor Marti Subrahmanyam, Professor Christina Wang, and Professor Wendy Jin for their roles in organizing the Honors Seminar. Additionally, my gratitude extends to Xinyi Yang for her diligent coordination of all the Friday meetings. These collective efforts have significantly enhanced my educational journey.

I. Introduction

Student grade inflation in the United States increased only gradually from the 1930s to the 1940s. In the 1960s, however, grades increased dramatically (Rojstaczer, 2003). Kuh and Hu (1999) argue that the average performance of all higher education institutions increased in the 1990s compared to the 1980s. At the universities studied by Sabot and Wakeman-Linn (1991), average grades increased from 2.38 in 1962 to 2.91 in 1985. As the standard deviation fell, grades were more concentrated at the top.

In empirical studies, scholars have provided a variety of factors that may boost grade inflation. Rosovsky and Hartley (2002) point out that the reason for the grade inflation in the United States in the 1960s was due to the incentive effect of the Vietnam War conscription on high grade. Kohn (2002) attributes the rise in student grades to improved teaching skills and frequent teacher-student communications.

Putting aside the influence of these exogenous factors, theorists have focused more on the influence of endogenous factors on the system. One view is that universities compete with other universities to secure their students' jobs, leading to grade inflation, and that top universities tend to inflate the grades of their students more than mediocre ones (Chan et al., 2007). Ostrovsky and Schwarz (2003) interpret grade inflation as a way for schools to add noise to the grade signal, boosting overall student wage earnings by lumping low-ability students together with high-ability students. A more classical view is that grade inflation has concentrated student grades at the top, making it hard to identify the best students. This hinders the positive matching between firms and students, which reduces the productivity of society and ultimately causes social welfare to suffer (Schwager, 2012). In Nordin et al. (2019), the source of social loss lies in the reduced learning motivation of students because of the high grade pulled by schools. In Chen et al. (2007), the loss comes from students' over-investment in education.

Many previous studies put performance inflation in a discrete game framework. For example, in Chan et al. (2007), students will receive either A or B, i.e., pass/fail. This makes sense (since the turn of the century, A has become the most common grade, with A proportion equal to that of the remaining letter grades combined) but given that students choose a combination of courses during college, the distribution of overall grades is still close to a continuum. This may imply that the discrete model setting still has room for improvement in generality. The fact that colleges add noise to students' grades is not directly based on students' real grades. Here I agree with Schwager (2012) that the confusion between high-ability students and low-ability students comes from the concentration of grades at the top.

This paper delves into the economic ramifications of grade inflation within higher education institutions and their strategic manipulation of grading policies. Starting from Spence's signaling model (1978) on an operational basis, it examines how universities employ grading as a strategic tool to enhance student enrollment and maximize revenue, effectively acting as monopolists in the educational market. It explores various equilibrium scenarios such as pooling equilibria, where students' abilities are indistinguishable based on grades alone, and separating equilibria, which accurately reflect student capabilities.

The analysis extends to the repercussions of these grading strategies on student welfare and the efficiency of the educational market, offering a critical view of the incentive structures that drive universities to inflate grades and the potential distortions it introduces into labor market signaling. This paper provides a comprehensive examination of the interactions between educational policies, university competition, and their collective impact on the labor market's functioning.

The remainder of the paper is structured as follows. Section II discusses the model setup and the pooling equilibrium at the ceiling. Section III examines the university as a monopolist, focusing on the incentives to inflate grades and the welfare of students. Section IV explores competition for enrollment between universities, including strategies for pooling or complete separation in student enrollment. Section V provides a discussion on the broader implications of the findings.

II. Grading and Ceiling Effect

A. Model Setup

There are three agents in the market: students, universities, and firms. Students have information about their ability $\alpha \in [0, \bar{\alpha}]$, which is also observed by universities. The distribution of student ability $F(\alpha)$ is public knowledge and assume to be non-convex. But for firms, they have no direct access to the ability of a particular student and can only obtain knowledge from the grades sent by universities.

Many firms in the market constitute a perfectly competitive situation. Every firm has the same production function $q = \alpha$ and makes zero profits. Based on their own ability, students will choose their own strategies τ to obtain a score $y \in [0, \bar{y}]$ and send it to firms. $\bar{y}$ is the upper limit of the score that a grading system can give. For firms, they have the best responding wage. Based on the assumption of zero profits, firms should pay wages equal to their belief $\hat{\alpha}$ about the ability of any student.

Students have the option to either enroll or opt out of university attendance. Should a student choose to opt out, their payment will be 0. Conversely, when a student decides to enroll in the university utilizing a signaling mechanism, the revenue to the student of sending grade y will be the wage $\hat{\alpha}$, and the cost is the effort required to obtain this grade. Therefore, its utility function is defined as follows:

(1) $$U(\alpha, \hat{\alpha}, y) = \hat{\alpha} - c(\alpha, y) - w,$$

where $c(\alpha, y) = max\left(0, \frac{y-\underline{y}}{\alpha}\right)$. $\underline{y} \geq 0$ is the guaranteed grade given by the university to the worst students. In other words, all students earn at least $\underline{y}$. w represents the tuition fee charged by the university to its students. This can also be seen as the one-time monetary cost that students pay to get into college as a signal transmitter. Here, tuition fees are assumed to be identical for all students.

It is evident that for students whose capabilities are below a threshold *w*, enrolling in the university will inevitably result in a loss, thus they will invariably exit and receive 0 profit. Specifically, when a student's ability exactly equals *w*, they are indifferent to the decision of whether to enroll or not. This constitutes the baseline for the signaling process.

The strategy for those who choose to enroll is

(2) $$\tau(\alpha) = \arg\max_{y} U(\alpha,\ \tau^{-1}(y), y).$$

In equilibrium, firms will know the correspondence between the strategies taken by students and the grades they obtain, that is, $\frac{d\hat{\alpha}}{dy} = \frac{d\alpha}{dy}$. Therefore, the FOC of students' utility with the respective of *y* reads

(3) $$\frac{d\alpha}{dy} - \frac{y}{\alpha} = 0.$$

Given the initial condition $\tau(w) = \underline{y}$, the mapping between grade y and ability α is

$$y - \underline{y} = \frac{1}{2}\alpha^2 - \frac{1}{2}w^2. \quad (4)$$

This function constitutes the basic condition in the market where scores map to abilities, and it satisfies the standard single-crossing condition.

B. Pooling Equilibrium at the Ceiling

There is a scope of application for the validity of this invertible relation. For high-ability students, there is an incentive to overinvest in education to achieve the highest grade and thus mix with students who have higher ability. Students at the tipping point will be indifferent between overinvesting and maintaining the status quo. At this point, a firm cannot discern student ability in the pooling equilibrium, so it will pay the wage expectation.

PROPOSITION 1: *Let $\alpha_{\overline{y}-\underline{y}}$ be the maximum ability that the current range of scores can cover. For any given distribution $F(\alpha)$ of student abilities, when the grading system does not cover the entire spectrum of student abilities, namely $\alpha_{\overline{y}-\underline{y}} < \bar{\alpha}$, there exists a critical ability level $\tilde{\alpha} < \alpha_{\overline{y}-\underline{y}}$. Students whose abilities exceed this critical point choose to achieve the highest score within the grading system, thereby resulting in a pooling equilibrium at the highest score level.*

PROOF:

See Appendix.

For students whose abilities are at the critical point $\tilde{\alpha}$, their utility function under the two scenarios—entering the pooling at the ceiling or accurately obtaining their true scores—is denoted as $U(\tilde{\alpha})$, which read

(5) $$U(\tilde{\alpha}) = \begin{cases} E(\alpha|\alpha > \tilde{\alpha}) - \frac{\overline{y}-\underline{y}}{\tilde{\alpha}} - w, \text{entering the pooling at the ceiling} \\ \tilde{\alpha} - \frac{\tau(\tilde{\alpha})-\underline{y}}{\tilde{\alpha}} - w, \text{accurately obtaining their true scores} \end{cases}.$$

Being at the critical point means the two choices above end up at same payoff. Combining this, one can derive the expression for $\tilde{\alpha}$.

Example 1: Given that student abilities $\alpha{\sim}Exp(\lambda)$ and tuition fee w equals 0, the wage offered by firms for reaching the score ceiling is $E(\alpha|\alpha > \tilde{\alpha}) = \tilde{\alpha} + \frac{1}{\lambda}$. By combining equation (5), one can obtain

(6) $$2\tilde{\alpha}E(\alpha|\alpha > \tilde{\alpha}) - \tilde{\alpha}^2 = 2(\overline{y} - \underline{y}) + w^2.$$

Taking the values of the wage and the tuition, the critical ability $\tilde{\alpha}$ reads

(7) $$\tilde{\alpha} = \sqrt{2(\overline{y} - \underline{y}) + \frac{1}{\lambda^2}} - \frac{1}{\lambda}.$$

This is less than the maximum value of ability that originally can be detected by the score, which is $\sqrt{2(\overline{y} - \underline{y})}$. Therefore, the relationship between grade and ability in equilibrium is deduced as

(8) $$\tau(\alpha) = \begin{cases} \frac{1}{2}\alpha^2 + \underline{y}, \ \alpha < \sqrt{2(\overline{y} - \underline{y}) + \frac{1}{\lambda^2}} - \frac{1}{\lambda} \\ \overline{y}, \ \alpha \geq \sqrt{2(\overline{y} - \underline{y}) + \frac{1}{\lambda^2}} - \frac{1}{\lambda} \end{cases}.$$

A representative image is as follows:

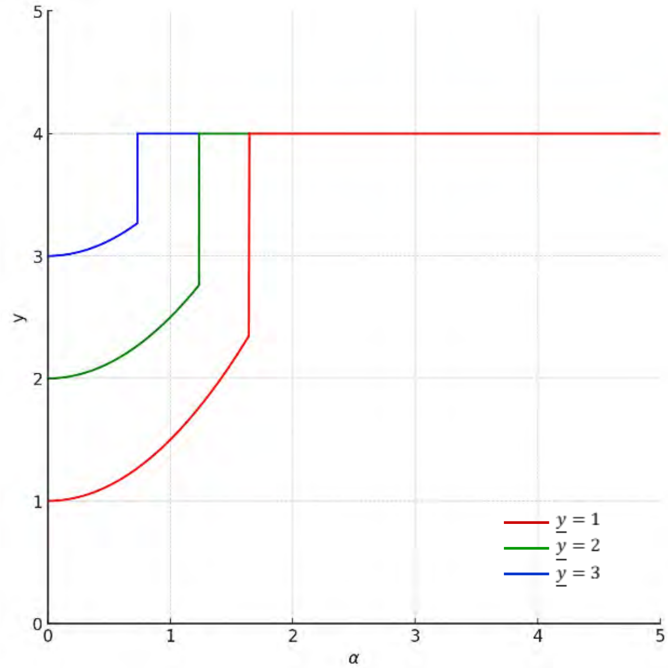

FIGURE 1. AN EXAMPLE OF $\tau(\alpha)$ IN EQUILIBRIUM

In Figure 1, λ is assigned a value of 0.5, and the maximum score $\overline{y}$ is assigned a value of 4. It is evident that by raising the minimum effort-free score, universities compress the actual detection range of scores as a signaling device. In the scenario of tuition-free education, the increase in the minimum score reduces the range of abilities that can be uniquely mapped by scores. In extreme cases, when the minimum score is raised to the maximum value, all students are leveled to the same position, and the score loses all its function as a signal.

In this model, the discontinuity in the mapping between student performance and student ability is inevitable due to the upper limit of grades. The single-crossing condition of total utility for students will ensure that those with higher abilities inevitably obtain higher utility. Hence, grade inflation, while reducing the cost of investment in learning, also enhances the benefits brought about by equilibrium pooling. Therefore, regardless of the distribution of student abilities, there will inevitably be students who benefit from the arbitrage opportunities brought about by the reduced detection interval of grades and choose to over-invest in education.

III. University as a Monopolist

A potential criticism of the setup in Example 1, where the university offers a free signaling service, may appear unrealistic. This chapter will formally analyze the university's objectives based on the scenario where there is only one university acting as a monopolist in the market.

Although universities today, whether public or private, are largely non-profit, this does not prevent tuition from being a significant source of income for universities. This income may be used to build laboratories and hire outstanding faculty and staff, or for-profit purposes. Therefore, defining the optimization objective of the university's grading policy as maximizing its tuition revenue is a reasonable choice.

For the convenience of subsequent deduction, one can simplify the previous representation of scores. When students choose their scores, their actual objective is the difference between the nominal score and the effort-free score provided by the university. Therefore, it is convenient to denote the university's score inflation tool, $\overline{y} - \underline{y}$, as $\overline{s}$. This represents the scaling space for scores. Consequently, the students' actual objective can also be rewritten as $s = y - \underline{y}$.

A. Incentive to Inflate Grades

Formally, the objective function of the university as a monopolist reads

(9) $$R(w, \overline{s}) = w \int_{A} dF(\alpha),$$

where A represents the set of abilities of all enrolled students. By the single-crossing condition and Property 1, given any pair $(w, \overline{s})$ where $\overline{s} > 0$, since the payoffs for students with relatively higher abilities are always greater than those for students with lower abilities, one can deduce the that for any student abilities α_i and α_j, if $\alpha_i < \alpha_j$ and $U(\alpha_i) > w$, then $U(\alpha_j) > w$. Therefore, under the given conditions of $(w, \overline{s})$, if $\alpha_i \in A$, then $\alpha_j \in A$. The critical case here is when $\alpha = w$; any student with $\alpha < w$ will choose to exit. This delineates $A = \{\alpha | \alpha \geq w\}$ when $\overline{s} > 0$. $R(w, \overline{s})$ then can be written as $w \int_{w}^{\overline{\alpha}} dF(\alpha) = w(1 - F(w))$.

In the scenario where $\overline{s} = 0$, the situation is particularly unique because all students can achieve the highest score with 0 effort, making the total cost and total profit of attending university homogeneous for students of any ability. Under this premise, for any tuition fee w, students will either all enroll or all exit. The complete withdrawal of students means 0 revenue for the university. Therefore, while experiencing complete grade inflation, the university does not raise tuition fees. The factor influencing the students' decision is the relationship between the overall expected ability of the students and the size of the tuition fees.

LEMMA 1: *When the university chooses complete grade inflation at* $\overline{s} = 0$, *it has to set the tuition fee* w *such that* $w \leq E(\alpha)$.

Based on Lemma 1, one can notice that when the university chooses $\overline{s} = 0$, its payoff $R(w, \overline{s}) = w$, where $w \leq E(\alpha)$. Given that all combinations of $(w, \overline{s})$ have been covered and discussed, the optimal strategy for the university monopolist can be derived.

PROPOSITION 2: *The optimal strategy for the university monopolist,* $(w*, \overline{s}*)$, *is uniquely* $(E(\alpha), 0)$. *That is, the university monopolist chooses full grade inflation while setting the tuition fee at* $E(\alpha)$.

PROOF:

When $w \leq E(\alpha)$, given any w, the condition $w(1 - F(w)) \leq w$ always holds. Therefore, $\overline{s} = 0$ is the optimal strategy under these circumstances. Among all combinations where $\overline{s} = 0$, the total tuition revenue for the school is maximized when $w = E(\alpha)$, reaching $E(\alpha)$. Hence, when $w \leq E(\alpha)$, the combination $(E(\alpha), 0)$ is the optimal strategy.

When $w > E(\alpha)$, only students whose abilities exceed w will choose to enroll, which implies $w \leq E(\alpha|\alpha \geq w)$. Multiply both sides with $(1 - F(w))$ and get

(10) $$w(1 - F(w)) \leq E(\alpha|\alpha \geq w)(1 - F(w)) = \int_w^{\overline{\alpha}} \alpha dF(\alpha).$$

Since $E(\alpha) = \int_0^{\overline{\alpha}} \alpha dF(\alpha)$, $E(\alpha) > \int_w^{\overline{\alpha}} \alpha dF(\alpha)$. Thus

(11) $$w(1 - F(w)) < E(\alpha) \quad w > E(\alpha).$$

This implies that when $w > E(\alpha)$, any combination $(w, \overline{s})$ is strictly less than $E(\alpha)$, which is the maximum payoff when $w \leq E(\alpha)$. Taken together, this proves that combination $(E(\alpha), 0)$ is the unique optimal strategy for the university monopolist.

B. Welfare of Students

The intuition behind the optimal strategy of the university monopolist is that it essentially captures all the surplus. Firstly, by offering the highest score with no effort required, the university motivates even students with zero ability to enroll, thus attracting a sufficiently large pool of students. Secondly, complete grade inflation minimizes the signaling cost that students need to bear. Lastly, setting tuition fees exactly at the expected level of student abilities makes all students precisely indifferent about whether to enroll or not. This strategy ensures that the university maximizes its revenue while maintaining maximum enrollment.

The analysis of the strategy of the university monopolist brings rather unfortunate news: the power of the monopolistic market is so strong that the welfare of the student group is reduced to zero. Students are simply bifurcated into two categories: those who enroll and those who do not. Although college students can earn a higher income compared to workers who do not attend college, they also have to pay high tuition fees, which equalizes the welfare levels between the two groups to the baseline.

IV. Competition for Enrollment

In a monopolistic academic environment where only a single educational institution exists, the utility derived by the most academically gifted students is adversely affected due to the phenomena of pooling with their lesser-performing peers. This pooling effect essentially dilutes the distinctiveness of top-tier students, obscuring their true academic prowess. To mitigate this diminution of their utility, these students are incentivized to seek alternative signaling mechanisms. These mechanisms serve to distinctly highlight their superior capabilities at a reduced cost, thereby preserving their competitive advantage in the academic and subsequent job markets. This dynamic introduces a powerful catalyst for competitive interactions between educational institutions, as they strive to attract these high-caliber students to enhance their institutional prestige and academic standing.

This chapter expands the analysis from a monopolistic scenario involving a single university to a market containing two universities, A and B. Before proceeding, some modifications and additional assumptions are necessary for the model. These universities are considered to be in the same market, appealing to a similar student population. They have identical objective functions denoted by $R(w,\overline{s})$. To more clearly analyze the role of grade inflation as a competitive tool between universities, certain simplifications must be made to enrich the model's analytical capacity. Specifically, the universities are assumed to be price takers; the tuition fees they charge are externally set and uniform across both institutions.

This assumption allows the focus to shift towards how these universities compete on non-price dimensions. By simplifying the tuition aspect, we can delve deeper into the strategic use of grade inflation in a competitive academic environment, examining how it impacts student enrollment decisions and the overall market dynamics between the two universities. Further, if we consider the heightened price sensitivity among consumers driven by increasing student loan debts and public scrutiny over the value of higher education, universities might find themselves constrained to adhere to prevailing market prices to attract and retain students.

The objective equations for the two universities are respectively

$$R_A(w, \bar{s}_A) = w \int_A dF(\alpha), \quad (12)$$

$$R_B(w, \bar{s}_B) = w \int_B dF(\alpha). \quad (13)$$

First, the two universities set their own grading ranges $\bar{s}_A$ and $\bar{s}_B$ respectively. The information of $\bar{s}$ is transparent and can be known by both students and firms. Then, students choose the university based on situations of $\bar{s}$ and send their grades through the university they choose. When prices are exactly the same, the competition between the two universities becomes a competition for the number of students' enrollment.

A. Pooling or Complete Separation in Student Enrollment

When deciding which university to enroll in, a student compares the payoffs offered by the two choices, represented as

(14&15) $$U_A(\alpha) = \begin{cases} E_A - \frac{\bar{s}_A}{\alpha} - w \\ \alpha - \frac{\frac{1}{2}\alpha^2 - \frac{1}{2}w^2}{\alpha} - w \end{cases}, \quad U_B(\alpha) = \begin{cases} E_B - \frac{\bar{s}_B}{\alpha} - w \\ \alpha - \frac{\frac{1}{2}\alpha^2 - \frac{1}{2}w^2}{\alpha} - w \end{cases}.$$

Specifically, if both universities provide the same payoff to a student, then the student is indifferent between the two and resorts to a random selection method such as flipping a coin to determine their choice of enrollment. This results in an equal probability distribution, with each university having a 50% chance of being selected by the student.

LEMMA 2: *In the spectrum of university grading policies, full grade inflation is strictly dominated.*

PROOF:

Suppose that in equilibrium, Ability expectations of students enrolled in the university with full grade inflation $E(\alpha|\alpha \geq \underline{\alpha}) > w$, where $\underline{\alpha}$ represents the minimum ability among students entering the university, it can be inferred that any student with ability $\alpha < \underline{\alpha}$ has a motive to enroll, since their utility of enrollment, $E(\alpha|\alpha \geq \underline{\alpha}) - w > 0$. Consequently, this does not represent an equilibrium state and creates a contradiction.

It follows that equilibrium in student enrollment occurs if and only if $E(\alpha|\alpha \geq \underline{\alpha}) = w$. However, under this scenario, any student with ability $\alpha \geq w$ would have a utility of 0. Competing universities with any $\bar{s} > 0$ can attract all students with abilities $\alpha \geq w$, in which case the university operating under full grade inflation would generate 0 total income, which reaches a minimum.

Lemma 2 primarily arises due to the competitive structure. Complete grade inflation is not a good choice for any student with abilities above w because they would suffer from free-riding by all lower-ability students. Thus, any university with $\bar{s} > 0$ becomes a better alternative. A university with complete grade inflation would inevitably exit the market with 0 income. This outcome implies that there will be no complete grade inflation in the equilibrium of the game involving university grading policies.

PROPOSITION 3: *Given a set of universities' grade inflation policies* $(\bar{s}_A, \bar{s}_B)$, *suppose* $\bar{s}_A > \bar{s}_B$, *there is a complete divergence from students above a certain ability* α_0. *That is,* $\exists \alpha_1 \in (\alpha_0, \bar{\alpha})$ *such that all students with ability* $\alpha \in (\alpha_1, \bar{\alpha})$ *choose university A, while the rest* $\alpha \in (\alpha_0, \alpha_1)$ *choose university B.*

When two universities implement identical grading policies, they essentially become perfect substitutes in the eyes of students. However, when universities offer different grading policies, a complete differentiation among students occurs. The essence of this phenomenon lies in the university that offers relatively lower grade inflation, which effectively raises the cost of signaling for students. This increased signaling cost becomes prohibitively high for students with lower abilities but remains tolerable for those with higher abilities.

As lower-ability students opt out due to these elevated costs, the implicit free-riding effect that would otherwise dilute the value of signals from high-ability students is reduced. This reduction in free riding makes it a more favorable option for high-ability students to choose the university with lower grade inflation over one with higher inflation. Such a scenario enhances the overall signaling effectiveness at the university with lower inflation, as it becomes a more targeted environment that better highlights and rewards true academic competence and potential. Thus, strategic differentiation in grading policies not only influences student choice but also potentially improves the alignment between students' abilities and the educational environments that best suit their needs.

B. Universities' Optimal Strategies

For any combination of university grading policies, students form a stable distribution of ability between the two schools, thereby generating a stable total revenue for each university. This stability provides universities with the opportunity to seek optimal grading strategies and to strategically game their grading policies. By manipulating their grading policies, universities can differentiate themselves in the market, attract distinct segments of the student population, and enhance their competitive positioning.

To further determine the precise grading strategies of universities at equilibrium, the distribution of student abilities needs to be explicitly specified. For simplicity, assume that student abilities $\alpha \sim U(0, \bar{\alpha})$. The total income for universities A and B can now be explicitly calculated as

(16) $$R_A(w, \bar{s}_A) = w[1 - (\frac{\alpha_1}{\bar{\alpha}} - \frac{1}{2}\frac{\alpha_0}{\bar{\alpha}}) - \frac{1}{2}\frac{w}{\bar{\alpha}}],$$

(17) $$R_B(w, \bar{s}_B) = w[(\frac{\alpha_1}{\bar{\alpha}} - \frac{1}{2}\frac{\alpha_0}{\bar{\alpha}}) - \frac{1}{2}\frac{w}{\bar{\alpha}}].$$

Here, $(\frac{\alpha_1}{\bar{\alpha}} - \frac{1}{2}\frac{\alpha_0}{\bar{\alpha}})$ represents the separate distribution of students among universities caused by their grading policies, while $\frac{1}{2}\frac{w}{\bar{\alpha}}$ represents the proportion of student population loss due to tuition pricing.

Under the uniform distribution assumption, the critical point α_1 at which students are completely separated is $\frac{\bar{s}_A - \bar{s}_B}{\frac{1}{2}(\bar{\alpha} - \alpha_0)}$. By performing calculations similar to those in Section 2, it can be determined that $\alpha_0 = \frac{2\bar{s}_B + w^2}{\alpha_1}$. Combining these two expressions, one can explicitly solve for α_1 and α_0 respectively as

(18) $$\alpha_1 = \frac{2\bar{s}_A + w^2}{\bar{\alpha}},$$

(19) $$\alpha_0 = \frac{2\bar{s}_B + w^2}{2\bar{s}_A + w^2}\bar{\alpha}.$$

By substituting equations (18) and (19) into equations (16) and (17), one can ultimately derive the total income of the universities as functions

(20) $$R_A(\bar{s}_A) = w[1 - (\frac{2\bar{s}_A + w^2}{\bar{\alpha}^2} - \frac{1}{2}\frac{2\bar{s}_B + w^2}{2\bar{s}_A + w^2}) - \frac{1}{2}\frac{w}{\bar{\alpha}}],$$

(21) $$R_B(\bar{s}_B) = w[(\frac{2\bar{s}_A + w^2}{\bar{\alpha}^2} - \frac{1}{2}\frac{2\bar{s}_B + w^2}{2\bar{s}_A + w^2}) - \frac{1}{2}\frac{w}{\bar{\alpha}}],$$

which are dependent on their grading policies. For better interpretation, note that $2\bar{s} + w^2$ represents the highest ability $\bar{\alpha}_{\bar{s}}$ that can be detected when the grading policy is $\bar{s}$. Define $\eta = \frac{\bar{\alpha}_{\bar{s}}}{\bar{\alpha}}$ as the detection capability of a university's grading system, then equations (20) and (21) can be rewritten respectively as

(20.1) $$R_A(\bar{s}_A) = w[1 - ({\eta_A}^2 - \frac{1}{2}\frac{{\eta_B}^2}{{\eta_A}^2}) - \frac{1}{2}\frac{w}{\bar{\alpha}}],$$

(21.1) $$R_B(\bar{s}_B) = w[({\eta_A}^2 - \frac{1}{2}\frac{{\eta_B}^2}{{\eta_A}^2}) - \frac{1}{2}\frac{w}{\bar{\alpha}}.$$

An observation from the game is that the lower the detection efficiency of a university's grading policy, the greater the potential income space it can generate; conversely, the detection efficiency of a competitor's grading policy has the opposite effect. This captures the inclination of universities to attract lower-ability students through grade inflation. At the same time, universities compete over the extent of grade inflation. A potential possibility is that when the detection efficiency of the grading policy of the university with relatively less grade inflation is sufficiently low, it provides a motivation for the university currently experiencing high grade inflation to improve its grading policy's detection efficiency.

PROPOSITION 4: *Under the condition of a uniform distribution of student abilities, the set of optimal grading policy combinations for universities, denoted as* $(\bar{s}_A *, \bar{s}_B *)$, *where*

(22) $$\bar{s}_A * = \frac{\bar{\alpha}^2 - 4w^2 + \bar{\alpha}\sqrt{\bar{\alpha}^2 + 8w^2}}{8}, \quad \bar{s}_B * \rightarrow 0.$$

The emergence of a duopolistic academic market, characterized by the presence of two universities, fundamentally alters the power dynamics, endowing students with a degree of bargaining power previously unseen. In such a scenario, a university will inevitably adopt a strategy focused on attracting top-tier students. This strategy is not merely preferential but essential, forming the cornerstone of the institution's competitive edge. A logical deduction from this competitive posture is that each university will endeavor to maximize its grade detection capabilities. This entails an expansion of the ability range that the highest grades can accurately signify, effectively raising the academic ceiling to capture the true extent of a student's capabilities.

However, this escalation in academic selectivity introduces a suite of complex challenges. In a bid to further amplify the expected wage outcomes for their graduates, an institution may resort to implementing stringent admissions criteria that disproportionately favor high-ability students. This approach could manifest in the direct exclusion of applicants perceived to have lower academic potential during the admissions process. Such a strategy, while ostensibly aimed at enhancing the quality of the student body and by extension, the institutional reputation, veers towards an alternative equilibrium. This equilibrium is characterized by heightened entry barriers, instituted as a means to preserve and enhance the quality of the student cohort. This selective admission process not only reflects the institution's commitment to academic excellence but also its strategic response to the pressures inherent in a competitive academic landscape.

V. Discussion

When a single university monopolizes the market, all students are treated indistinguishably, resulting in a utility of 0 for everyone. However, when competition emerges in the market, universities begin to differentiate and stratify. One university attracts students with higher abilities, while another gathers those with lower abilities. However, students near the lowest ability threshold appear in everywhere. It can be observed that the policy of grade inflation is mitigated by competition, yet this dramatically alters the distribution of grades in the market. At the ceiling, there still exists a large number of students achieving perfect scores. For universities with less grade inflation, due to their higher ability student body, the proportion of students congregating at the ceiling may even be greater. Overall, however, the number of students achieving perfect scores is still reduced compared to the monopolistic state.

Is grade inflation a balloon that lifts overall social welfare, or just a bubble that creates opportunities for free-riding? The results of this paper may not point definitively in either direction. It should be said that it itself constitutes another kind of signal reflecting the overall condition of students within a university: University A, having less grade inflation than University B, thus indicates higher overall student ability at University A.

It is important to note that this paper only discusses the competition between two universities and simplifies many other factors present in reality. Firstly, there is a certain degree of variability in the relationship between grades and abilities, and the extent of this variability also affects students' decisions in signaling. Secondly, universities themselves may have brand effects, thus even in the presence of competition, they possess some monopolistic power, and therefore the tuition fees they charge are subject to adjustment. These are some limitations of the current study, but they also become directions for future research.

APPENDIX

PROOF OF PROPOSITION 1:

To prove the existence of the critical point $\tilde{\alpha}$, it is sufficient to demonstrate that $\tilde{\alpha}$ ensures that any student with an ability above $\tilde{\alpha}$ will choose to achieve the ceiling score. Formally, this requires showing $\forall \alpha \in (\tilde{\alpha}, \alpha_{\overline{y}-\underline{y}}), E(\alpha|\alpha > \tilde{\alpha}) - \frac{\overline{y}-\underline{y}}{\alpha} - w > \alpha - \frac{\tau(\alpha)-\underline{y}}{\alpha} - w$, where the left side represents the utility of a student choosing the ceiling score, and the right side represents the utility of a student honestly choosing their effort.

If $\tilde{\alpha}$ exist, $then\ E(\alpha|\alpha > \tilde{\alpha}) - \frac{\overline{y}-\underline{y}}{\tilde{\alpha}} - w = \tilde{\alpha} - \frac{\tau(\tilde{\alpha})-\underline{y}}{\tilde{\alpha}} - w$. This gives

(A1) $$E(\alpha|\alpha > \tilde{\alpha}) = \frac{1}{2}\tilde{\alpha} + \frac{\alpha_{\overline{y}-\underline{y}}^{2}}{2\tilde{\alpha}}.$$

Let $h(\alpha) = \frac{1}{2}\alpha + \frac{\alpha_{\overline{y}-\underline{y}}^{2}}{2\alpha}$. By checking its First Order Condition (FOC), one can derive

(A2) $$h'(\alpha) = \frac{1}{2} - \frac{\alpha_{\overline{y}-\underline{y}}^{2}}{2\alpha^{2}}.$$

When $\tilde{\alpha} < \alpha < \alpha_{\overline{y}-\underline{y}}$, $h'(\alpha) < 0$. Hence $h(\alpha)$ is decreasing on $(\tilde{\alpha}, \alpha_{\overline{y}-\underline{y}})$. This implies $\forall \alpha \in (\tilde{\alpha}, \alpha_{\overline{y}-\underline{y}}), \frac{1}{2}\alpha + \frac{\alpha_{\overline{y}-\underline{y}}^{2}}{2\alpha} < \frac{1}{2}\tilde{\alpha} + \frac{\alpha_{\overline{y}-\underline{y}}^{2}}{2\tilde{\alpha}} = E(\alpha|\alpha > \tilde{\alpha})$, which shows the sufficient inequality.

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