The Marriage Proposal Game

by

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Abstract

I analyze the strategic dynamics of the Marriage Proposal Game, a signaling game with alternating decision roles and structure. I show that under the basic framework, equilibria involve no delay, as impatient ("low") types always have an incentive to mimic patient ones, preventing separation. This result is in line with the literature of classic signaling games in the case where signals are costless. Extending the basic game, I introduce a Marriage Market Structure, where separating equilibria arise by giving players the opportunity to be rematched with new partners. In equilibrium patient ("high") types receive higher payoff in expectation that in the simple form of the game, since they use the market mechanism as a signaling device. Although framed metaphorically as a model of marriage, the insights could extend to settings like corporate joint ventures or strategic negotiations, where the two parties participating have an equal role of a sender and receiver.

Key Words: *Marriage Proposal, Game Theory, Market Design, Signaling Games*

Preface

The starting point of the chosen topic was my general interest in economic theory, and more specifically topics in bounded rationality. Inspired by models such as Glazer's and Rubinstein's model of persuasion with boundedly rational agents [2], I initially had the idea of extending these form of models into a model where both agents take the role of the speaker and the listener, something I had in mind as a model of "bilateral persuasion". Throughout the course of me writing the thesis the topic slightly changed, and ended up having no elements of bounded rationality. This was partially owning to me being at an early stage in my studies, leading my advisor to encourage me to pursue more "traditional" avenues.

I see this more as an "attempt" of doing research on my own in the field of economic theory, or as my advisor described it, as a "creative exercise". The paper presented below is complete and written in the style of other papers in economic theory, however I acknowledge that there are no extremely novel and groundbreaking ideas. My advisor had a policy of not providing specific directions for my paper, which encouraged me to think creatively and develop my own ideas. While this approach pushed to think about research independently, it may have also constrained the final outcome. I was well aware of these trade-offs from the beginning of this thesis, and I remain grateful for the opportunity to explore various themes in economic theory under my advisor's guidance.

1 Introduction

A couple just got together. They both want to get married in their life, however they are possibly not sure if their current partner is the right one for them. At each time period one of them can choose to Wait (W) and let the other player decide in the next time period, propose to the other player (Pr), or break up (B). Once a player has proposed, the other player can either choose Yes (Y) (ending the game in agreement - and getting married) or No (N) (ending the game in disagreement - and breaking up). If the game goes on indefinitely, it is considered a disagreement. Therefore there are only two possible results of the game, and there is no partial agreement. This alternating structure is shown in figure 1.

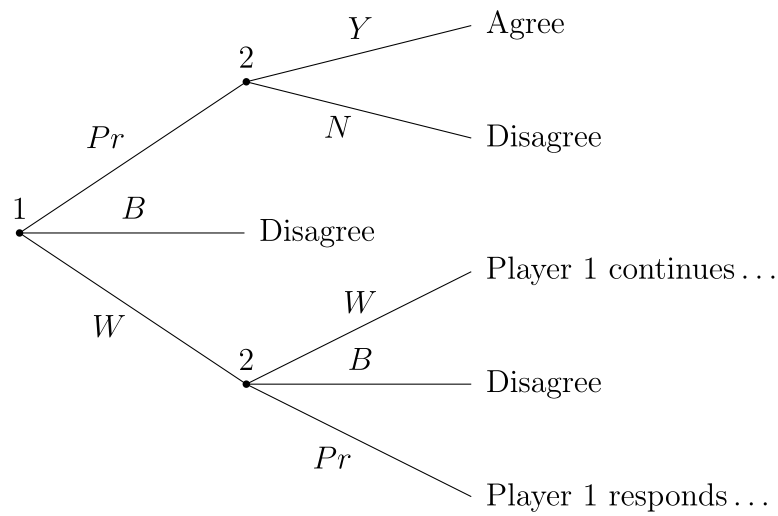

Figure 1: The Marriage Proposal Game

This model broadly falls under the family of signaling games. There are two different types that each player might fall under (for the time referred to as “low" and “high" type), and it is always in the benefit of each player to signal themselves as the high type. In contrast to traditional signaling games (e.g. Spence’s Job Market signaling [8]) that typically involve a clear distinction between a sender and a receiver, in my setting both players possess private information about their own type and alternate roles, each having opportunities to act and respond according to the alternating structure.

To accommodate for the two players alternating between the roles of the sender and the receiver, a non-cooperative type bargaining structure is adopted (see [6]). Much like in models of bargaining, at each time period one of the players has the chance to make a proposal (with the difference that there is no partial agreement), which the other player can either accept of reject. The most important difference is that here there is no partial agreement, and there are only two possible outcomes of the game.

This strategic situation vaguely resembles the kind of interaction in the War of Attrition games (see for example [3]), in the sense that in both my model and the War of Attrition, players engage in a waiting game where each must decide whether to act immediately or delay in hopes of achieving a better outcome. However, in my model, once a player makes a move, the other player determines the outcome by either accepting or rejecting the proposal, rather than the game ending immediately. Most importantly, my model assumes no difference in the payoffs between the player who acts first (proposes) and the one who follows (responds). Therefore waiting here can only benefit players as a form of costless signaling device for the patient types, and not in terms of directly affecting their payoff.

In the most basic form of the marriage proposal game which is described in Section 2 and analyzed in Section 3, players play the game described in the first paragraph of the introduction (formalized later). The main question of interest is whether with the given alternating structure, it is possible for the high types to signal their identity and result in a pooling or separating equilibrium. Under the specific assumptions made in Section 2 the answer is negative. Proposition 3 shows that when waiting is not explicitly costly (it is costly only through a discounting factor in the players' utilities), it is impossible for the high types to separate themselves, while Propositions 2 and 1 show that in the only two equilibria of the game the expected utility of the high types is exactly 0.

To resolve this, in Section 4 I propose a market extension of the marriage proposal game (much like the market extension of bargaining models, see [5]), where after breaking up players have the chance to be matched again with other players that have remained single after the first time period. Even the basic version that I analyze with one additional time period is enough to (strictly) improve the outcomes of the high types in expected terms. High types can now use breaking up as a signaling device, and receive strictly positive payoffs from reaching an agreement in the second time period.

The rest of paper is organized as follows. Section 2 describes the basic framework of the Marriage Proposal Game. Section 3 analyzes the basic form of the game, with two results about existence of Perfect Bayesian Equilibria, and one result that shows that there can be no separation between the two types (no delay). In Section 4 I present the market extension of the game, Section 5 concludes.

2 Basic Framework

There are two players, labeled by $\{1, 2\}$. Their type is drawn from the set $\{Patient(P), Impatient(I)\}$ independently with uniform probability, and the set of possible types is denoted by $\Theta = \Theta_1 \times \Theta_2$. Each player perfectly knows his own type, but not the type of the opponent. Time is discrete and denoted by $t \in \{1, 2 \ldots\}$. The set of actions for player 1 at odd times is $\{Propose(Pr), Wait(W), Break\ Up(B)\}$, while at even times it is $\{Yes(Y), No(N)\}$ and vice versa for player 2. The set of possible results of the game are $R = \{Agree, Disagree\}$, where an agreement is reached only if a player proposes and the other player accepts.

The preferences of each player depend on the two types $\theta \in \Theta$ and the terminal history of the game. An impatient type only cares about marrying fast, regardless of who his partner is. On the other hand, a patient type wants to ensure his partner is also patient, and only in that case he is getting positive utility from getting married. Formally the utilities of the two players are define over the set $\Theta \times R$ as follows:

Assumption 1. *The utility of reaching an agreement at time t for the patient type is:*

$$u_P^t = \begin{cases} (\delta_P)^{t-1}, & \textit{if opponent is patient,} \\ -(\delta_P)^{t-1}, & \textit{if opponent is impatient.} \end{cases}$$

while for the impatient type is:

$$u_I^t = (\delta_I)^{t-1}$$

where $0 < \delta_I < \delta_P < 1$. In all cases the utility of a disagreement is 0.

Denote by H^t the set of feasible (non-terminal) histories of the game at time t, and let h^0 denote the empty history. Further let A_i^t be the set of feasible actions for player i at time t. The strategies and the beliefs of both players are formally defined below.

Definition 1. *A (behavior) strategy for a player 1 of type θ_1 is an infinite sequence of functions $f_{\theta_1} := (f_{\theta_1}^1, f_{\theta_1}^2, \ldots)$, where $f_{\theta_1}^t : H^{t-1} \to \Delta(A_1(h^{t-1}))$ defined over all non-terminal histories, where $\Delta(A_1(h^{t-1})$ is the set of probability measures over the set of feasible strategies for player 1 after history h^{t-1} has occurred. Similarly define $g_{\theta_2} := (g_{\theta_2}^1, g_{\theta_2}^2, \ldots)$ to be the strategy of a player 2 of type θ.*

Definition 2. *Player i's beliefs is a function $\mu_i : H^{t-1} \times \Theta_i \to \Delta(\Theta_{-i})$. That is, each* *player's belief assigns a probability that his opponent's type is $\theta_{-i} \in \Theta_{-i}$ conditional on his own type and the time period that has been reached. I assume this is well defined for all non-terminal histories h^{t-1} and all times t, and that beliefs are common knowledge between the two players.*

The solution concept used in the next section is that of Perfect Bayesian Equilibrium (PBE). The formal definition following Fudenberg and Tirole [1] is as follows:

Definition 3. ***Perfect Bayesian Equilibrium.*** *A Perfect Bayesian Equilibrium (PBE) is a strategy profile (f, g) for each possible type of players $1, 2$ and beliefs μ such that:*

1. *For every non-terminal history h^{t-1}, the strategies of both players are a Bayesian Equilibrium[1] of the continuation game given their beliefs at time t.*

2. *Bayes's rule is used to calculate beliefs whenever possible.*

3. *Beliefs are independent of the type of each player. That is, for all players i, $\mu_i(\theta_i|h^{t-1}) = \mu_i(\theta_i'|h^{t-1})$ for all θ_i, θ_i' and all histories h^{t-1}.*

PBE first of all imposes a form of subgame perfection. Players are required to act optimally after each history of the game given their beliefs at that point, ruling out many Bayesian Equilibria that can occur by having players play strictly dominated actions on off equilibrium paths.

PBE also restricts the beliefs that the two players can have, especially on off equilibrium paths by requiring that they are equal regardless of their type[2]. When beliefs are allowed to differ for different types of the same player many sorts of equilibria

[1] Heuristically, in a Bayesian Equilibrium each player is maximizing expected utility conditional on their type, given their opponents strategy.

[2] Equivalently, one can demand that beliefs at each time period only depend on the actions of the opponent up to that point.

can arise. To that direction, the definition of Perfect Bayesian Equilibrium above, imposes that beliefs are independent of the type of each player. These restrictions are proven to be (under conditions satisfied here) equivalent to the consistency requirement imposed by Kreps [4] in the definition of sequential equilibria, without requiring the use of trembles to generate beliefs.

3 Perfect Bayesian Equilibria

The following tie-breaking rules simplify the analysis of the game.

Assumption 2. *When a player is indifferent between waiting and ending the game or proposing, he will either choose to end the game or propose. When a player is indifferent between accepting and rejecting a proposal he will accept it.*

The following (trivial) lemma is frequently used below.

Lemma 1. *In a PBE a patient type will accept a proposal at time t with probability 1 if and only if $\mu_i(P|h^{t-1}) \geq \frac{1}{2}$, and will always reject it otherwise. An impatient type will always accept a proposal with probability 1.*

Since at all times t there is only one possible non-terminal history (the one where players have only chosen to wait), I write $\mu_i(\theta_{-i}|h^{t-1}) := \mu_i^t(\theta_{-i})$.

3.1 Existence

The following two propositions are about the existence of equilibria with no delay. In both cases they rely on generating beliefs such that player 2 is certain that if he observed his opponent waiting, then he is facing an impatient player. This 'forces' patient players to either propose or break up immediately in the first period.

Proposition 1. *There exists a PBE where impatient players make a proposal immediately, which is accepted if only if the other player is impatient, and patient players stop the game immediately.*

Proof. Let the beliefs for 2 in the second period be $\mu_2^2(I) = 1$ (i.e. player 2 believes that player 1 is certainly impatient when he observes delay). Given these beliefs it has to be that in the second period a patient type stops the game immediately, while an impatient type will make a proposal, that is only accepted by the impatient types of player 1. It follows immediately that the strategies where at $t = 1$ player 1 of patient types stops the game and impatient types of player 1 propose, which is accepted only by the impatient types of player 2 is a PBE. □

Proposition 2. *There exists a PBE where everyone gets married immediately.*

Proof. Similarly to above let $\mu_2(I|\{W\}) = \mu_2(I|\{Pr\}) = 1$. An identical argument as above verifies that the strategies where both types of player 1 propose immediately and both types of player 2 accept is a PBE. □

3.2 Uniqueness

It turns out that these are the only two PBE of the game. That is, there can never be delay in equilibrium. The "intuitive" result, where the patient type waits for a few periods and then gets married while the impatient type proposes immediately does not exist in this setting. Impatient types always find it optimal to "disguise" themselves as patient in the cases where there is no delay, and this makes waiting pointless for the patient types.

Assumption 2 is crucial for the result here. If you allow patient players to wait when their expected utility is 0 there can be a PBE where both players wait until a specific time period and then agree at that point by assigning pessimistic beliefs on the off-equilibrium paths. The formal proposition is given below:

Proposition 3. *(No Delay) There is no Perfect Bayesian Equilibrium where player* 1 *waits with positive probability in the first time period.*

The logic of the proof is as follows. For delay to occur it has to be that a patient type gets (in expectation) strictly positive utility by waiting at time $t = 0$. This in turns implies that it has to be at least one decision node where patient players propose more often that impatient players, but whenever that happens it is optimal for impatient players to deviate to proposing at the same rate as patient players.

Proof. Assume by contradiction that there exists a PBE where delay occurs with positive probability. Let p_t^I be the probability an impatient player proposes at time t (sim. p_t^P). In an equilibrium with delay it has to be that for at least one t where player 2 moves, $p_t^P > p_t^I$ - call τ the first time period for which this happens - since otherwise the expected utility of waiting in the first time period for a player 1 of patient type is 0 and he would immediately stop the game.

But now consider the continuation game starting at time τ. Since this the first time period for which $p_t^P > p_t^I$ it follows that $\mu_i^\tau(I) = \mu_i^\tau(P)$. As long as $p_t^I \leq p_t^P$, the expected utility of an impatient player is equal to $p_t^I \delta^{\tau-1} + (1 - p_t^I)\mathbb{E}[\text{Wait}]$ where $\mathbb{E}[\text{Wait}] \leq \delta^\tau$. So a deviation to $p_\tau^I = p_\tau^P$ is optimal for the impatient type, which is a contradiction. $\square$

4 The Marriage Market

A natural extension to the game above is to consider a (decentralized) market for marriage, where after a breakup there is the possibility of being matched with a new partner. This is similar to the market extension of the bargaining models in Osborne and Rubinstein [5] (henceforth OR), adjusted to the structure and story of the Marriage Proposal Game. My formulation below is in line with a story in which there is no anonymity. People can observe the past actions of everyone's relationships, and players' actions toward their partners at each time period might influence their future outcomes.

We first need to make a clear distinction between potential players 1 and 2. Intuitively, potential players 1 can be thought as heterosexual males, while players 2 as heterosexual females. Let N_1 denote the number of males entering the game at $t = 1$, and N_2 the number of females. I assume that there is only entry in the first period and that no new agents enter the market at some later time period (similar to model B in OR) which in the context of marriage proposal can be interpreted as people restricting themselves to marrying people of a certain age, and thus there can be no new entry in the marriage market for them.

Similarly to before, players might be of either patient of impatient type, and their utilities are defined in the same way as above. I let N_1^P denote the number of males of patient type and N_1^I the impatient, with $N_1^P + N_1^I = N_1$ (sim. for player 2). In contrast to OR, here player 1 always makes the first move. This can be justified as a form of social norm, where in most societies men are expected to propose. In line with the assumption made before about the types being realized with uniform probability, I am going to assume that for the players entering the market $N_1^P = N_1^I = N_2^P = N_2^I$, which also guarantees that all players are always matched with someone. The matching technology is uniformly random; for example at $t = 1$ the probability that a randomly chosen player is matched with a patient type of player 2 is equal to $\frac{N_2^P}{N_2}$.

For a start I am assuming that there are only two time periods, after which if a player remains unmatched their utility is 0. Summarizing, the structure of the market is shown in figure 2. At each time period $t \in \{1, 2\}$ the following events take place:

1. **Matching:** Single agents from Population 1 and Population 2 are randomly paired.

2. **Marriage Proposal Game:** Each matched pair engages in the Marriage Proposal Game as defined in Section 2, with the difference that there is no option to stay in the current matching.

3. **Outcomes:**

   - **Agree (A):** The pair marries, exiting the market.
   - **Disagree (D):** In the first time period both players become single and re-enter the matching pool, while in the second time period the game ends and both players remain unmatched.

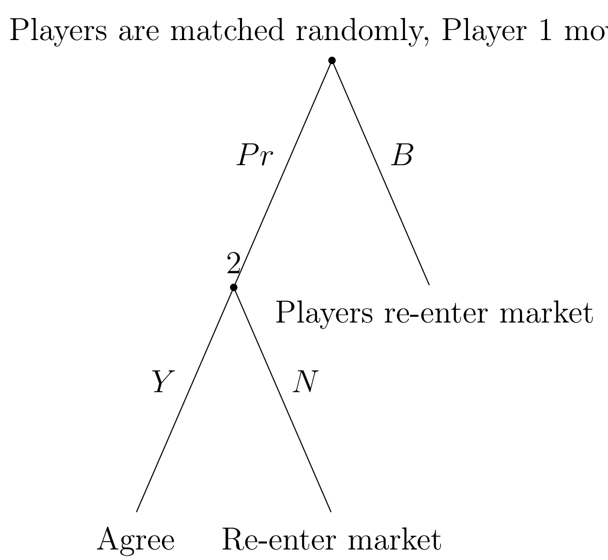

Figure 2: The Marriage Market

In contrast to most models in OR, the strategies of both players are allowed to depend on the past actions of their opponents (the history of their opponents in their previous encounters). This is different than the assumption of semi-stationarity often made in models in the literature, where agents' strategies may only depend on the history of events within a match.

4.1 Market Equilibrium

Definition 4. *A market equilibrium is a strategy profile for each of the $N_1 + N_2$ players, such that each player's strategy is optimal in the sense of Perfect Bayesian Equilibrium at every point at which the player has to make a choice, given the assumption that the unobserved actions of other players align with their equilibrium strategies.*

Essentially when players make a decision in the first time period, they are going to assume that everyone else will act according to their equilibrium strategies. Therefore, when evaluating the expected utility of waiting to be matched again in the second round, they will take into account how many players of each type are expected to be single after the first period based on their equilibrium strategies. For simplicity, I am assuming that for the impatient player, $\delta_I < \frac{1}{2}$. While this might not be necessary, if one does not make this assumption the calculations of expected utility become very convoluted, given that one has to consider all possible realizations of the random matching process in the first time period.

Proposition 4. *Suppose that $\delta_I < \frac{1}{2}$. Then there exist a market equilibrium where:*

- *Impatient men propose in the first time period, which is accepted only by impatient women.*

- *Patient men break up in the first time period.*

- *In the second time period the remaining impatient men propose, while patient men's strategy depends on the number of patient women remaining.*

Proof. Recall that an impatient player's utility for marrying at time t is $(\delta_I)^{t-1}$. Clearly since $\delta_I < \frac{1}{2}$ an impatient player 1 will propose immediately, since the expected utility of proposing is $\frac{1}{2}$ and the maximum possible utility from waiting is δ_I.

Consequently, in period 1, only "impatient man, impatient woman" pairs marry. This removes some impatient women from the pool, leaving a strictly higher fraction of patient women for period 2, so in equilibrium the patient men will break up in the first round. Following the matching in the beginning of the second period, the type of all men is perfectly known, while for women the type is only known for the patient ones that rejected a proposal in the first time period.

Therefore at $t = 2$, impatient men will propose, and the proposal will be accepted if and only if the woman is impatient. For patient men, if the share of unknown patient women is higher than the number of unknown impatient women, he will propose no matter what. Otherwise, he is only going to propose when he is matched with a woman he knows to be patient. $\square$

In light of this proposition, it is clear that even the (trivial) extension of the game gives a strictly higher payoff to Patient types. More specifically, the expected utility of a Patient type of Player 1 here is $\frac{\delta_P}{4}$, as opposed to 0 in the basic version of the game. Under the assumption that $\delta_I < \frac{1}{2}$, adding more time periods in the market would further increase the expected utility of patient players.

5 Final Comments

This paper should not be read as a literal representation of the process of marriage or relationships but rather as a framework that extends some of the analysis and intuition of the strategic interaction in signaling games. Marriage is employed here as an intuitive metaphor that allows the insights to be communicated more effectively. The use of such relatable narratives aligns with my advisor's view that economic models are better viewed as didactic stories, much like fables or fairytales (see [7]).

The alternating decision structure of the game emphasizes the iterative nature of signaling and decision-making under incomplete information, a feature present in various real-world strategic settings. While traditional models like Spence's signaling theory assume rigid roles of sender and receiver, this model alternates these roles.

A possible takeaway from this model is that inefficiency in signaling when waiting has no explicit cost extends to the specific alternating structure introduced here, leading to the impossibility of separating equilibria under the game's basic framework. The introduction of a marriage market highlights the transformative power of extending the model's scope to allow for post-breakup rematching, which serves as an additional layer of signaling. This extension reflects how market mechanisms can resolve inefficiencies in bilateral settings, an insight that is in line with broader discussions in economics about the value of decentralized systems in improving welfare outcomes.

As mentioned above this model can be describe in more general terms compared to the marriage example. One could for example frame this as a model of two companies that are deciding whether to participate in a joint venture or not, where there are "good" and "bad" types of companies. The Marriage example was chosen for its simplicity and intuitive nature, and to make it clear that the model is not meant to be taken literally.

Ultimately in my view, this study is just one example of how economic theory can abstract and formalize complex social phenomena, and help give some intuition about the way humans act in some situations. While the model is basic and bound to the constraints that come with me being at an early stage in my studies, the general aim is not to prescribe or strictly predict behaviors, but to provoke discussion on the structures underlying human interaction, offering a starting point for intelligent conversations about the design and efficiency of social arrangements.

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Appendix A Social Norms on Marriage Proposal

Social norms play a significant role in shaping how individuals approach marriage proposal; in many cultural contexts, there is a common expectation that men are more likely to propose, though this norm is not universal and has evolved over time. This not meant to be a normative but rather a positive statement, reflecting patterns observed in certain societies. In this section, I present a (failed) attempt to explore how incorporating such norms into the model can impact the strategic dynamics of the game.

One way to formalize a traditional norm within the game is to assume that only player 1 (he) can propose, while player 2 (she) can either wait or break up. This is an assumption that I partially made in Section 4, and below I explain why it has no effect in the basic version of the Marriage Proposal Game.

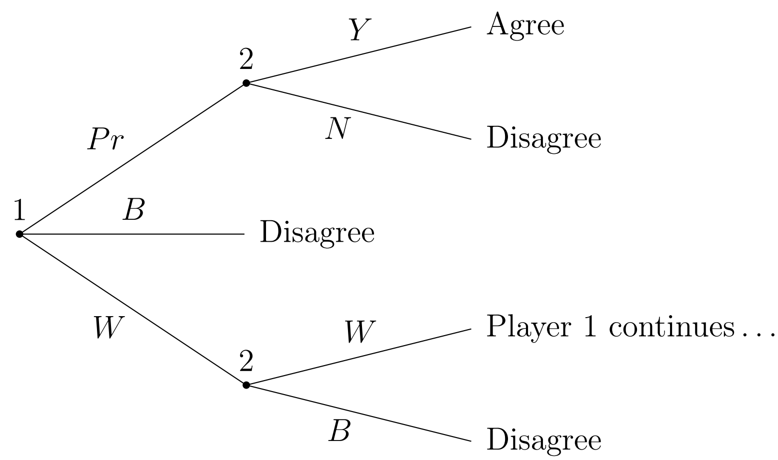

Figure 3: Alternating Structure with Social Norms

The definitions of strategies remain as before, it is only the set of possible actions for player 2 that has now changed. Propositions 1 and 2 still go through with a slight change of the argument. Again if you let $\mu_2^2(I) = 1$ both equilibria exist, since any deviation to waiting for player 1 would lead to impatient types of player 2 immediately stopping the game. Therefore patient types of player 1 have nothing to gain by waiting (the maximum utility you can get from waiting is 0 since all patient players are immediately gone), and impatient types of player 1 are only reducing their utility by waiting to reach an agreement at a later time period.

Same holds true for the uniqueness result. There is delay only if there is some time period where the patient player proposes more often than the patient one, and whenever that happens it is optimal for the impatient player to deviate to proposing at the same rate as the impatient one.

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