Sample Placement Test for Honors Calculus and Honors Linear Algebra I

Exercise I

1. Solve the equation $3\log(2x) + 1 = y$, where the unknown is x. In particular, say how many solutions the equation has, depending on y.

2. Same question for the equation $2e^{3x} - 1 = y$.

Exercise II

Say whether the following statements are true or false.

1. If $\dfrac{f(x) - f(y)}{x - y} > 0$ for all $x \neq y$, then f is an increasing function.

2. If a function f has $f(x) > 0$ and $f(y) < 0$ for some x and y, then there is a z such that $f(z) = 0$.

3. If f is increasing on $\mathbb{R}$, then there is no M such that $f(x) \leq M$ for all $x \in \mathbb{R}$.

4. If f is an odd function on $[-1, 1]$, then the equation $f(x) = 0$ has a solution.

5. 1 is an element of the set $[0, 2]$.

6. $\{1\}$ is an element of the set $[0, 2]$.

7. $[0, 1]$ is an element of the set $[0, 2]$.

8. $[0, 1] \cup [0, 2] = [0, 2]$.

9. $[0, 1] \cap [0, 2] = [0, 1]$.

Exercise III

1. **(True or False)** If $P(x) = x^4 + 3x^2 + x$ and $Q(x) = 2x^4 + 5x^2 + 2x$, then $Q(x) \geq P(x)$ for all $x \in \mathbb{R}$.

2. **(True or False)** A polynomial can have infinitely many roots.

3. If $P(x) = x^7 + x^6 - 8x^5 - 4x^4 + 21x^3 - 18x + 6$, what is the sum of the roots of P?

4. Factorize the polynomial $2x^5 - 6x^3 + 4x$.

Exercise IV

1. Graph the function $f(x) = e^{-x} + 1$.

2. Graph the function $f(x) = \log(x + 1)^2$.

Exercise V

1. Solve the inequality $|x - 2| + |x - 3| < 3$.

2. Solve the inequality $x^2 + x - 3 > 3$.

Exercise VI

Three men named Paul, Jack, and Bill, discuss.

- The first one says "My name is Paul, and the second person is Bill."

- The second one says "The first person says the truth."

- The third one says "The first man is Paul."

You know that only one person speaks the truth. Who is who?

Exercise VII

1. How many solutions does the equation $\cos x = \sin x$ have in $[-\pi, \pi]$?

2. What are the solutions to $\cos x = 1/2$ in $[-2\pi, 2\pi]$?

3. Write $\cos(a + b) + \cos(a - b)$ as a product.

4. Write $\cos x + \cos y$ as a product.