Sample Placement Test for Honors Calculus
and Honors Linear Algebra I

Exercise I

1. Solve the equation $3 \log(2x) + 1 = y$, where the unknown is $x$. In particular, say how many solutions the equation has, depending on $y$.

2. Same question for the equation $2e^{3x} - 1 = y$.

Exercise II

Say whether the following statements are true or false.

1. If $\frac{f(x) - f(y)}{x - y} > 0$ for all $x \neq y$, then $f$ is an increasing function.

2. If a function $f$ has $f(x) > 0$ and $f(y) < 0$ for some $x$ and $y$, then there is a $z$ such that $f(z) = 0$.

3. If $f$ is increasing on $\mathbb{R}$, then there is no $M$ such that $f(x) \leq M$ for all $x \in \mathbb{R}$.

4. If $f$ is an odd function on $[-1, 1]$, then the equation $f(x) = 0$ has a solution.

5. 1 is an element of the set $[0, 2]$.

6. $\{1\}$ is an element of the set $[0, 2]$.

7. $[0, 1]$ is an element of the set $[0, 2]$.

8. $[0, 1] \cup [0, 2] = [0, 2]$.

9. $[0, 1] \cap [0, 2] = [0, 1]$.

Exercise III

1. (True or False) If $P(x) = x^4 + 3x^2 + x$ and $Q(x) = 2x^4 + 5x^2 + 2x$, then $Q(x) \geq P(x)$ for all $x \in \mathbb{R}$.

2. (True or False) A polynomial can have infinitely many roots.

3. If $P(x) = x^7 + x^6 - 8x^5 - 4x^4 + 21x^3 - 18x + 6$, what is the sum of the roots of $P$?

4. Factorize the polynomial $2x^5 - 6x^3 + 4x$. 
Exercise IV

1. Graph the function \( f(x) = e^{-x} + 1 \).
2. Graph the function \( f(x) = \log(x + 1)^2 \).

Exercise V

1. Solve the inequality \(|x - 2| + |x - 3| < 3\).
2. Solve the inequality \(x^2 + x - 3 > 3\).

Exercise VI

Three men named Paul, Jack, and Bill, discuss.

- The first one says “My name is Paul, and the second person is Bill.”
- The second one says “The first person says the truth.”
- The third one says “The first man is Paul.”

You know that only one person speaks the truth. Who is who?

Exercise VII

1. How many solutions does the equation \( \cos x = \sin x \) have in \([-\pi, \pi]\)?
2. What are the solutions to \( \cos x = 1/2 \) in \([-2\pi, 2\pi]\)?
3. Write \( \cos(a + b) + \cos(a - b) \) as a product.
4. Write \( \cos x + \cos y \) as a product.