Mathematical Approaches to Real Time Bidding Strategy (RTB) for Advertising Campaigns for Demand Side Platforms (DSP)

by

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1 Abstract

Real Time Bidding (RTB) Strategy is playing a more and more important role in digital advertising, in which it helps to win expected numbers of advertising campaigns with a minimum cost for Demand Side Platforms (DSP). Every impression offered through Sell Side Platforms (SSP), can be regarded as a new type of supply since it is unpredictable and unable to be inventoried. Therefore, the core of my thesis would be, how should we build a mathematical optimization model to better catch every impression with the lowest cost, given that the supply is uncertain and cannot be stored. The whole thesis would come with a general model with factors in both dimensions and depths, then focuses on a relaxation on certain constraints, and finally find out the solutions based on the previous analysis by setting bounds for both the cost and the numbers of impressions. Moreover, we will define some variables which have certain realistic meanings to further interpret the results extracted from our model. Some typical mathematical domain knowledge such as theory of probabilities, statistics, convex optimization, etc. will be applied to solve the research questions.

2 Acknowledgments

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3 Introduction

Nowadays, many applications on mobile phones will show the advertisement banners for several seconds after users open it. This is a new kind of business which has appeared recently. Here, I am going to illustrate the players who are engaging in this game. First of all, the market, where all the spaces for advertisement display are bought or sold, is called the Ad Exchange. Usually, Sell Side Platforms (SSP), the supplier of the spaces such as the owners of the mobile apps, are responsible for arranging the spaces in a proper way such that every space can be sold efficiently. Similarly as what happens on the supply side, the advertiser such as companies who create ads to promote their goods or services, will find a Demand Side Platform (DSP) to help them fulfill their advertising campaign goals in a certain period of time. For example, company A may pay a DSP $50 to win 1000 ads spaces in one week. Certainly, in real life business, some more details will be given out, such as the target customer segmentation, the time periods or the regions the company prefer, etc.

The opportunity to display an ad on an App is referred to as an impression. Until recently, accompanied with the appearance of mobile Ad Exchange, the supply of impressions on Apps is becoming a commodity. Moreover, there are no longer contracts through a long-time period and each opportunity to display an ad is auctioned off on a mobile Ad Exchange. This great change happened on the supply side also altered the demand side of the industry. DSP can now directly buy supplies from exchanges rather than accessing the supplies via SSP. As a result, DSP should provide access to supplies via Ad Exchange and their key expertise should be bidding intelligently to win impressions on Ad Exchange. In this case, Real Time Bidding strategy becomes an effective way to accurately predict the supply as well as win it at a low price if this strategy can be notated as some mathematical models or algorithms.

In order to show the markets in an integral way, figure 1 shows the mechanisms of how RTB works with SSP and DSP. Figure 2 shows the companies that are doing these kinds of business in this industry. The digital Ads market has been growing rapidly in the past few years and we can see that many hi-tech and algorithm companies are playing crucial roles in this field. Therefore, I believe, the optimization of RTB strategy is a topic that is worth being discussed.

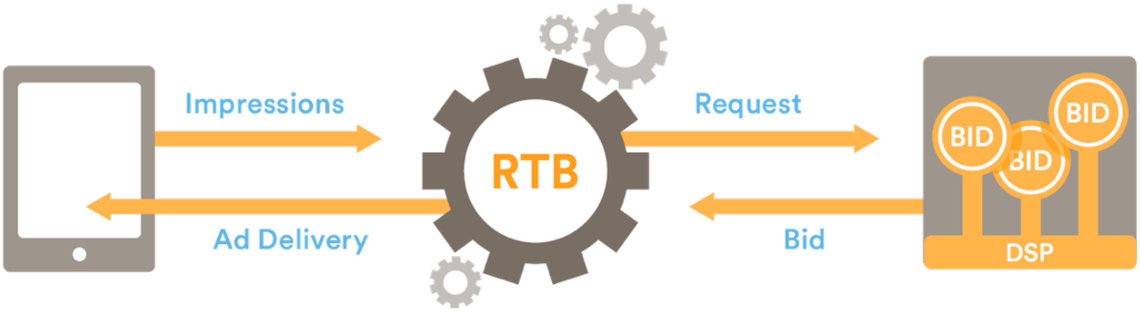

Figure 1: Mechanisms of How RTB Works with SSP and DSP

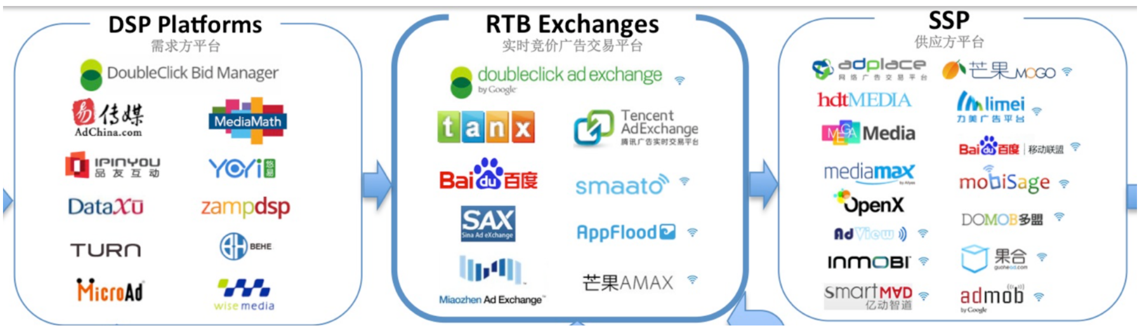

Figure 2: Industry Development in China

4 Literature Review

According to the latest IAB Internet Advertising Revenue Report released by the industry trade group and prepared by PwC US, digital advertising revenues in the United States for the first half of 2017 surged to an all-time high of $\$40.1$ billion. The development is so rapid that a 40 percent increase from $\$15.5$ billion in half-year 2016 and far surpassing the $\$8.2$ billion reported just two years ago in HY 2015.

As discussed in the paper written by Balseiro et al.(2014), traditionally, an advertiser would directly buy placements and sign contracts with the publisher, i.e. the owner of the digital space. And most of the deals would be reserved over a specific time span. The click-through rate would be measured as a quality of those placements. But things gradually changed when Ad Exchange came. According to Muthukrishnan (2017), another mechanism has began to form: an auction will run as soon as the publisher post an ad slot. Advertisers will post bids and the winner will successfully display their advertisements. All the above happens as soon as a user opens an app or a web page and he/she is waiting for the content to be loaded.

Therefore, under such situation, the supply of impressions is uncertain and may be varied over time compared to the classical auction models. New methodologies are needed to conquer these difficulties. Many scholars have addressed various approaches to provide new methods to solve this problem. For example, Ghosh et al.(2009) proposed a practical bidding mechanism to achieve reasonable performance by creating the randomized bidding rules. Turner (2012) focuses on the allocation of guaranteed targeted display advertising by proposing a quadratic objective optimization formulation and algorithms.Another scholar, Chen(2017), has complemented more by introducing the dynamically intertwined incentives of the publisher because of the instantaneous benefit and the threat of contractual penalty for nondelivery.

As mentioned in a survey written by Korula et al. (2016), there are 3 important elements in such kind of problems: targeting, volume and price. Moreover, inspired by 2 recent studies, the main work this thesis has done is to formulate this strategy to be a non-linear programming problem which contains the key elements mentioned above. Ciocan and Farias (2012) consider a class of dynamic allocation problems with unknown and volatile demand. We refered to their methods of modeling the uncertain demand and adjusted it to some extent to better match our whole model. Aseri et al. (2017) has modeled this question into a quite decent and clear way.

The construction of our model is mainly inspired by their ways of separating the problem into a static one and a dynamic one. The creative and initial parts we added on are: First, we simplified the location variable by changing it to be a vector which contains all the info (graphical, preference, age, etc.) of every potential user. This would make our targeting to be more specific and efficient. Secondly, we added the CTR (click through rate) into the model because we believe this would be a good measure which reflect the quality of each bidding and might be useful if we could record it.

5 Quantification of relevant variables

5.1 Supply Side

The Ad Exchange runs a real-time second-price auction every time an impression comes. There are multiple exogenous advertisers and DSPs participating in the auctions of every space that Ad Exchange offers. First, we should quantify the impressions that meet the advertiser's targeting criterion (such geographic location, age group of the reviewer, tastes, and interests obtained from her browsing history) as $W = [w_1, w_2, ..., w_n]$, which is a multi-dimensional vector stores various features of a specific user. In order to ensure that all the impressions would arrive separately, we set 2 level units of time to cut every slot of time. $S = \{1, 2, ...s\}$ represents every day, $I = \{1, 2, ...i\}$ represents every time block in which there would be only one impression comes in each i. Finally, we set $T = \{1, 2, ...t\}$ where $t = s \cdot i$.

Suppose impressions arrive at the Ad Exchange according to a Bernoulli Distribution with parameter $q_{i,w}$, which denotes that the probability of an impression from W arrives for $t \in T_{s,i}$. Similarly, we use $g_{i,w}$ to denote the probability of an impression from W won at $t \in T_{s,i}$ and will be clicked by the user.

5.2 Demand Side

As discussed in the previous section, DSP now faces a situation where it has made firm delivery commitments to customers but its supply of impressions is uncertain. Therefore, we would like to quantify the supply using the mathematical knowledge in Theory of Probabilities. We consider that over a fixed time horizon $[0, T]$. The advertiser signed a contract with a DSP including 4 variables (T, M_c, M_c^{click}, R) where T refers to the length of time, M_c refers to the targeting number of impressions for a specific campaign c, M_c^{click} refers to the targeting number of click-throughs of these impressions, and R refers to the budget that the adviser is willing to pay. Here, campaign $c \in C$, which $C = \{1, 2, ..., c\}$ and $c = 2^w - 1$ for $w \in W$. Thus, at time T, the DSP needs to win M_c impressions, M_c^{click} click-throughs to get revenue R from the advertiser. Given other constraints and conditions, we would like to optimize his RTB strategy by involving these 4 main variables.

In brief, here is a summary of all the crucial variables:

- W: vector, general features of the potential viewer = $(w_1, w_2, w_3, \ldots, w_n)$ e.g. w_1: sex, w_2: age, w_3: location, etc.
- C: set, various campaigns = {1, 2,…, c} $c = 2^n - 1$
- S: set, days = {1, 2,…, s}
- I: set, time blocks = {1, 2,…, i}
- T: set, the planning horizon consists of the set of time slots = {1, 2,…, t} $t = s \cdot i$
  - i.e.ensure there would not be 2 impressions arrive simultaneously
- $q_{i,w}$ = prob. of an impression from W arrives for $t \in T_{s,i}$ [related to j]
- $g_{i,w}$ = prob. of an impression from W won at $t \in T_{s,i}$ and then be clicked [related to k]
- win curve: $p_{i,w}(b)$: $[0, b_{i,w}^{max}] \rightarrow [0, 1]$ where b = bid price
- bid curve (win curve^{-1}): $b_{i,w}(x)$: $[0, 1] \rightarrow [0, b_{i,w}^{max}]$ where x = winning probability
- $f_{i,w}(x) = x \cdot b_{i,w}(x)$ i.e. winning probability for an impression comes from w, time block i · expected cost

- A set of $U[0, 1]$ random variables (a standard and convenient mathematical device)
- $\{U_t^{win} : t \in T\}$ and $\{U_t^{alloc} : t \in T\}$ denote two sets of mutually i.i.d., $\{U_t^{imp} : t \in T\}$ independent
- $j_{t,c}$ = # of impressions won by campaign c before the start of time slot t (U_t^{imp}: Ber($q_{i,w}$))
- $k_{t,c}$ = # of click-throughs reached till time t for campaign c (U_t^{click}: Ber($g_{i,w}$))
- J_t : set = set of state vectors j_t in time slot t
- π: a certain policy
- $x_{t,w}^{\pi}(j_t)$ = win prob. for this impression which comes from w in time slot t
- $y_{t,w,c}^{\pi}(j_t)$ = allocated prob. to campaign c given that policy π has won this impression

- **Conceptually obvious equations involving above variables would be discussed in the constraints

6 Modeling and Optimization

6.1 General Settings

As discussed previously, the impressions are auctioned in real time on an Ad Exchange. The platform, and also other advertisers or companies acting on behalf of advertisers, bid for these impressions. The highest bidder wins the impression and follows the rules in a second-price auction. Clearly, the higher the bid, the greater the probability of winning an impression. Therefore, $a\ win\ curve$ can be easily set as a function $p_{i,w}(b) : [0, b_{i,w}^{max}] \to [0, 1]\ for\ w\ \in W, i \in I$, which indicates the probability of winning that impression by bidding an amount b. As a result, the inverse function of the $win\ curve$ can be called as a $bid\ curve$: $b_{i,w}(x) : [0, 1] \to [0, b_{i,w}^{max}]\ for\ w \in W, i \in I$, where x is the winning probability. After all these settings, it is clear that the subjective function in this optimization problem would be: $f_{i,w}(x) = x \cdot b_{i,w}(x)$, which shows the whole cost, i.e. the winning probability for an impression comes from $w \in W$, time block $i \in I$, and then times the expected cost. For any time period in T, let $j_{t,c}$ denote the number of impressions won by campaign c before the start of time slot t. J_t then is the set of all feasible state vectors in time slot t. Similarly, let $k_{t,c}$ denote the number of click-throughs reached till time slot t for campaign c. Now we design a bidding strategy π to maximize the DSPs' expected profit subject to the 4 variables (T, M_c, M_c^{click}, R).

To be more specific, $x_{t,w}^{\pi}(j_t)$ is the winning probability for this specific impression. Because we have considered the fact that a DSP would have multiple clients, the allocation should also be a variable built in our model. Consequently, we set $y_{t,w,c}^{\pi}(j_t)$ to be the allocated probability to campaign c given that policy π has won this impression. We note that $x_{t,w}^{\pi}(j_t) = \Sigma_{c \in C}\ y_{t,w,c}^{\pi}(j_t)$, for $\forall\ j_t \in J_t$.

Since our model will involve various probabilities, we use a standard and convenient mathematical device here. We use a set of $U[0,1]$ random variables to fit the probabilities in. Then, we would have 3 separate U: U_t^{imp}, U_t^{win}, U_t^{alloc}. $U_t^{imp} = 1$ only when the impression comes from the specific w and take value 0 at other times. Then we say that an impression is won by policy π if and only if $U_t^{win} \leq x_{t,w}^{\pi}(j_t)$. Further, if this impression is won, then it is allocated to campaign c if $U_t^{alloc} \in [\Sigma_{\hat{c}=1}^{c-1} y_{t,w,\hat{c}}^{\pi}(j_t)/x_{t,w}^{\pi}(j_t),\ \Sigma_{\hat{c}=1}^{c} y_{t,w,\hat{c}}^{\pi}(j_t)/x_{t,w}^{\pi}(j_t)]$. All these relations would be used as our constraints.

Up to now, all the preparations are done and we can use all the parameters discussed above to generate a convex problem to optimize.

6.2 Analysis of a Static Model

We first deal with a static model in which it is impossible that many impressions have been won at early periods of T and then the it may start bidding low on subsequent impressions. We denote this situation as $\mathscr{P}_{static}(M, \alpha, M^{click}, \alpha^{click})$. Here, α and α^{click}, representing the probabilities of completing the requirements from clients, are values that are close to one. Then, we quantify our objective function as f^{π}:

$$f^{\pi} = \Sigma_{s \in S}\ \Sigma_{i \in I}\ \Sigma_{w \in W} \mathbb{E}_{U_{t-1}}[q_{i,w} f_{i,w}(x_{t,w}^{\pi}(j_t^{\pi}(U_{t-1})))]$$

Therefore, the problem $\mathscr{P}_{static}(M, \alpha, M^{click}, \alpha^{click})$ can be written as follows:

$$[\mathscr{P}_{static}(M, \alpha, M^{click}, \alpha^{click})] :$$

$$\mathbf{min} f^{\pi} \tag{1}$$

subject to

$$\Sigma_{c\in C}\ y^{\pi}_{t,w,c}(j) = x^{\pi}_{t,w}(j), \quad \forall t\in T, j\in J_t, w\in W \tag{2}$$

$$j^{\pi}_{t+1,c}(U_t) = j^{\pi}_{t,c}(U_{t-1}) + \Sigma_{x\in W}\mathbf{1}(U^{imp}_t = w)\ \mathbf{1}(U^{win}_t \le x^{\pi}_{t,w}(j^{\pi}_t(U_{t-1})))$$
$$\times\,\mathbf{1}\left(\frac{\Sigma_{\hat{c}=1}^{c-1} y^{\pi}_{t,w,\hat{c}}(j^{\pi}_t(U_{t-1}))}{x^{\pi}_{t,w}(j_t(^{\pi}(U_{t-1})))} < U^{alloc}_t \le \frac{\Sigma_{\hat{c}=1}^{c} y^{\pi}_{t,w,\hat{c}}(j^{\pi}_t(U_{t-1}))}{x^{\pi}_{t,w}(j_t(^{\pi}(U_{t-1})))}\right), \forall t\in T, c\in C, u_t\in U_t \tag{3}$$

$$\mathbb{P}[j^{\pi}_{t_c+1}(U_{t_c}) \ge M_c] \ge \alpha \ \ \forall c\in C \tag{4}$$

$$k^{\pi}_{t+1,c}(U_t) = j^{\pi}_{t,c}(U_{t-1}) + \Sigma_{x\in W}\mathbf{1}(U^{imp}_t = w)\ \mathbf{1}(U^{win}_t \le x^{\pi}_{t,w}(j^{\pi}_t(U_{t-1})))$$
$$\times\,\mathbf{1}\left(\frac{\Sigma_{\hat{c}=1}^{c-1} y^{\pi}_{t,w,\hat{c}}(j^{\pi}_t(U_{t-1}))}{x^{\pi}_{t,w}(j_t(^{\pi}(U_{t-1})))} < U^{alloc}_t \le \frac{\Sigma_{\hat{c}=1}^{c} y^{\pi}_{t,w,\hat{c}}(j^{\pi}_t(U_{t-1}))}{x^{\pi}_{t,w}(j_t(^{\pi}(U_{t-1})))}\right)\mathbf{1}(U^{click}_t = w),$$
$$\forall t\in T, c\in C, u_t\in U_t \tag{5}$$

$$\mathbb{P}[k^{\pi}_{t_c+1}(U_{t_c}) \ge M^{click}_c] \ge \alpha^{click} \qquad \forall c\in C \tag{6}$$

6.3 A Relaxation of the Static Model

We here introduce a new variable $\beta_c \ge 0$, which represents a certain number of impressions, $in\ expectation$ for each campaign $c\in C$. Similarly, we have another variable β^{click}_c which represents the ideal numbers of click-throughs. Formally, we denote a new problem by:

$$\mathscr{P}^{E}_{static}(\beta)$$

Therefore, the new problem would be written as follows:

$$\mathscr{P}^{E}_{static}(\beta):\begin{cases}\textbf{min} f^{\pi} \\ \textbf{subject to } (2)\text{-}(6) \\ \mathbb{E}[j^{\pi}_{t+1,c}(U_{t_c})] \geq \beta_c, & \forall c \in C \\ \mathbb{E}[k^{\pi}_{t_c+1}(U_{t_c}) \geq M^{click}_c] \geq \beta^{click}_c & \forall c \in C\end{cases}$$

Then, we can apply the Markov's inequality: $P(X \geq a) \leq \frac{\mathbb{E}(X)}{a}$ to find a suitable β and write it into the formula. Since $\mathbb{P}[j^{\pi}_{t_c+1}(U_{t_c}) \geq M_c] \geq \alpha \;\; \forall c \in C$, it is intuitive to rewrite the formula: $\mathbb{E}[j^{\pi}_{t+1,c}(U_{t_c})] \geq \alpha M_c, \forall c \in C$. Similarly, we have another formula to rewrite: $\mathbb{E}[k^{\pi}_{t+1,c}(U_{t_c})] \geq \alpha^{click} M^{click}, \forall c \in C$.

As a result, $\mathscr{P}^{E}_{static}(\alpha M_c)$ is a relaxation of problem $\mathscr{P}_{static}(M, \alpha, M^{click}, \alpha^{click})$. We can then begin to solve $\mathscr{P}^{E}_{static}(\beta)$ for any β.

7 A Near-Optimal Solution of the Static Model

7.1 Lower Bound of the Required Impressions

Now that we have constructed a static model here, we can simply set a linear relationship between β_c and β_c^{click} by multiplying a click-through rate $g_{t,w}$ for every t in T, every w in W (mentioned before in Section 2.1). Our main approach is that we want to find a lower bound which defines the minimum required amount of Ads, thus we can find an upper bound for the maximum cost paid by DSPs.

First, we noticed that the number of impressions won for each campaign c over every time slot has a binomial distribution. Further, given that large number of time slots over the whole time span, the total number of impressions assigned to campaign c is approximately normally distributed with mean $\Sigma_{s=1}^{K_c}\Sigma_{i\in I}\hat{p}_{s,i,c}^{\beta}$ and variance $\Sigma_{s=1}^{K_c}\Sigma_{i\in I}\hat{p}_{s,i,c}^{\beta}(1-\hat{p}_{s,i,c}^{\beta})$. Therefore, we can have the formula which represent the relationship between the probability with the required numbers of Ads:

$$\Sigma_{s=1}^{K_c}\Sigma_{i\in I}\hat{p}_{s,i,c}^{\beta} - z_\alpha(\Sigma_{s=1}^{K_c}\Sigma_{i\in I}\hat{p}_{s,i,c}^{\beta}(1-\hat{p}_{s,i,c}^{\beta}))^{1/2} = M_c, \qquad c\in C$$

Since the probability that an impression arrives in any location in a given time slot is extremely small, the value of $1-\hat{p}_{s,i,c}^{\beta}\approx 1$ for any β. Hence, the equation above would be deducted as:

$$\Sigma_{s=1}^{K_c}\Sigma_{i\in I}\hat{p}_{s,i,c}^{\beta} - z_\alpha(\Sigma_{s=1}^{K_c}\Sigma_{i\in I}\hat{p}_{s,i,c}^{\beta})^{1/2} = M_c, \qquad c\in C$$

From the quantification of different variables, we can see that, for the optimal solution, $\Sigma_{s=1}^{K_c}\Sigma_{i\in I}\hat{p}_{s,i,c}^{\beta} = \Sigma_{s=1}^{K_c}\Sigma_{i\in I}\Sigma_{w\in W_c}q_{i,w}\hat{y}_{s,i,w,c}^{*}(\beta) = \beta_c$. Up until now, the equation has been deducted to:

$$\beta_c - z_\alpha\beta_c^{1/2} = M_c$$

By solving the above formula, we obtain that $\beta_c = (2M_c + z_a^2 + \sqrt{(2M_c + z_a^2)^2 - 4M_c^2})/2$. Because $\alpha \approx 1$ in our settings, hence $\beta_c = M_c + z_a\sqrt{M_c}$. This form reminds us of the news-vendor model, in which case, $z_\alpha\sqrt{M_c}$ is the safety stock for campaign c. This formula defines the lower bound of the required impressions we should achieve in our model.

Practically, for a campaign $c \in C$, the ratio $\frac{\beta_c}{\alpha M_c}$ implies the percentage of the additional number of impressions needed by our policy. Let $\gamma = \mathbf{max}_{c \in C}[\beta_c^*/(\alpha M_c)] = \mathbf{max}_{c \in C}[(1/\alpha) \cdot (1 + z_\alpha/\sqrt{M_c})]$. Therefore, γ is the maximum additional percentage of impressions needed by our policy. For example, if $\gamma = 1.02$, it means our model would acquire at most 2 percentage additional impressions.

7.2 Upper Bound of the Estimated Cost

In order to find the upper bound of the estimated cost generated by the constraints above, we define a new variable called $\psi_{i,l}(x)$, where:

$$\psi_{i,w}(x) = \frac{x f'_{i,w}(x)}{f_{i,w}(x)}, \forall i \in I, w \in W, x \in [0, 1]$$

The realistic meanings are as follows: The numerator of the expression on the right-hand side of the above equation is the marginal expected cost at win probability x, while the denominator is the marginal expected cost at win probability x if the expected cost function were a linear function with a slope $f_{i,w}(x)/x$. In other words, $\psi_{i,w}(x)$ us a measure of how fast the expected cost function changes relative to a (hypothetical) linear cost function.

Since $\alpha \in [0.95, 0.99]$ and M_c is of the order of 100,000, we have $\gamma x_{max} \ll 1$. We proceed under the assumption that $\gamma x_{max} \ll 1$, and define $\bar{\psi} = \mathbf{max}_{x \in [0, \gamma x_{max}]} \mathbf{max}_{i \in I, w \in w} \psi_{i,w}(x)$. Deducting from the definition above, $\bar{\psi}$ is the maximum rate at which the expected cost function increases relative to a linear cost function.

Assume that $\bar{\psi} < \gamma/(\gamma - 1)$. This assumption is easily satisfied in realistic problem instances. For example, in the discussion in "Procurement Policies for Mobile-Promotion Platforms" (Aseri et al. 7) , we will see that $\bar{\psi}$ is an order of magnitude smaller than $\gamma/(\gamma - 1)$. As a result, we have found the upper bound of the cost:

$$\frac{f^{\pi(\beta^*)}}{\mathbf{Opt}(M, \alpha)} \leq \frac{1}{1 - ((\gamma - 1)/\gamma)\bar{\psi}}$$

Now we present a real life example: Let $M_c = 150,000$ for all c, x_{max} = 0.05 and α = 0.99. Then, γ = 1.016. Let $\bar{\psi} = 1.4$. Then, the bound $\frac{1}{1-((\gamma-1)/\gamma)\bar{\psi}} = 1.0225$. From our model, this indicates that the total cost is at most 2.25 percentage higher than that of the optimal policy for $\mathscr{P}_{static}(M, \alpha)$.

7.3 The Role of Click-Through Rate

As the CTR is implemented in the same way as the winning rate, we can easily apply the same logic to find the lower bound of the required click-throughs as well as the upper bound of the estimated cost.

Therefore, $\beta_c^{click} = M_c^{click} + z_{\alpha^{click}}\sqrt{M_c^{click}}$ represents the lower bound of the required click-throughs we should achieve in our model. As a result, in order to check the realistic application, let $\gamma^{click} = \mathbf{max}_{c\in C}[\beta_c^{*click}/(\alpha^{click}M_c^{click})] = \mathbf{max}_{c\in C}[(1/\alpha^{click}) \cdot (1 + z_{\alpha^{click}}/\sqrt{M_c^{click}})]$. Therefore, γ^{click} is the maximum additional percentage of click-throughs needed by our policy. For example, if $\gamma^{click} = 1.05$, it means our model would acquire at most 5 percentage additional click-throughs.

Next, let us find the upper bound of the cost if we need to achieve a certain number of click-throughs in a advertising campaign. By using the same methodology, we know that the $\bar{\psi} = \mathbf{max}_{x\in[0,\gamma x_{max}]}\mathbf{max}_{i\in I,w\in w}\psi_{i,w}(x)$. Hence, here is the upper bound of the cost for the required click-throughs:

$$\frac{f^{\pi(\beta^*)}}{\mathbf{Opt}(M^{click}, \alpha^{click})} \leq \frac{1}{1 - ((\gamma^{click} - 1)/\gamma^{click})\bar{\psi}}$$

This upper bound could present a range that can guide the DSP sides' decisions when signing contracts with their clients.

Moreover, with the results of two groups of variables: β_c, β_c^{click}, $\frac{1}{1-((\gamma-1)/\gamma)\bar{\psi}}$ and $\frac{1}{1-((\gamma^{click}-1)/\gamma)^{click}\bar{\psi}}$, DSP can consider more deeply about the profitability of a certain advertising campaign and can also make much wiser decisions in allocating any won impression. For example, empirically, they should first consider those advertising campaigns which have lower $\frac{1}{1-((\gamma-1)/\gamma)\bar{\psi}}$ and $\frac{1}{1-((\gamma^{click}-1)/\gamma)^{click}\bar{\psi}}$ because when DSPs finishes the campaign requirements, their cost would not be too higher and it would be better and faster for them to cover their original cost.

8 Discussion and Potential Improvements

As this thesis is mainly a theoretical approach, what is the value of it and how can it be referenced by this industry?

As discussed before, digital advertising industry is a field which embeds huge potential, and is already developing rapidly thanks to some advanced technology. Recently, there are even some papers discussing the application of deep reinforcement learning in real-time bidding. For example, Zhao et al. (2018) have discussed to apply deep reinforcement learning for sponsored search real-time bidding and Cai et al. (2017) also applied reinforcement learning in developing more advanced real-time bidding strategies. As a result, I think in the future, more display advertising strategy will rely much on such kind of artificial intelligence instead of some classical mathematical modeling methodologies.

However, digital advertising is still a relatively newborn industry. It would be costly for the machine learning technique to realize its functions because of the scarcity of available data. Under such circumstances, the mathematical approach can show its predictive power through its rigorous logic and scientific structure. As this thesis has shown so far, we get a range to control the cost for a set of campaigns starting from a optimization problem.

But still, we want to discuss the disadvantages of this thesis. It is not a completed one because we do not have a chance to test our models by using real-world datasets. Although we presented the realistic meaning of some variables we have set, it should be tested via a real digital advertising data sets to check whether the market also works in way as we assumed.

I think, in the future, other mathematical or engineering knowledge should be focused on finding optimal solutions to some questions such as allocations of different impressions onto various advertisement campaigns, etc. I believe such type of strategic development can help more DSPs to fulfill their advertising requirements much efficiently even for a huge set of clients.

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